# AN INVESTIGATION OF
# THREE-DIMENSIONAL PROBLEM SOLVING AND LEVELS OF THINKING
# AMONG
# HIGH SCHOOL GEOMETRY STUDENTS

---

A Project Report

Presented to

The Graduate Faculty

Central Washington University

---

In Partial Fulfillment

Of the Requirements of the Degree

Master of Arts in Teaching

Mathematics

---

By

Josh Wheatley

August 2011

CENTRAL WASHINGTON UNIVERSITY

Graduate Studies

We hereby approve the project thesis of

Joshua Anton Wheatley

Candidate for the degree of Master of Arts in Teaching, Mathematics

APPROVED FOR THE GRADUATE FACULTY

_________ ______________________________________

Dr. Mark Oursland, Committee Chair

_________ ______________________________________

Dr. Michael Lundin

_________ ______________________________________

Dr. Dominic Klyve

_________ ______________________________________

Dean of Graduate Studies

# ABSTRACT

## AN INVESTIGATION OF THREE-DIMENSIONAL PROBLEM SOLVING AND LEVELS OF THINKING AMONG HIGH SCHOOL GEOMETRY STUDENTS

By

Josh Wheatley

August 2011

The purpose of this study was twofold. First, the researcher wanted to investigate the level of thinking of students in Geometry in solving three-dimensional problems. Secondly, the researcher wanted to investigate the affect of the van Hiele theory on measurement of Surface Area and Volume. The van Hiele level theory served as the basis for the investigation. The researcher's intent was to supplement the traditional geometry curriculum with a series research-based activities aimed at increasing the level of thinking among his students. A convenience sample of two geometry classes taught by the researcher allowed the researcher to compare the results of traditional instruction with those of the curriculum supplement. Results indicated a significant effect of the activities on the level of reasoning and achievement of the students.

ACKNOWLEDGMENTS

"The fear of the LORD is the beginning of knowledge…"

The Bible, Proverbs 1:7

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

# CHAPTER 1

# INTRODUCTION

High school geometry students have difficulty transitioning from two-dimensional to three-dimensional geometry with particular difficulty with volume and surface-area concepts (Bako, 2003; Gutierrez, 1992; Kenney & Kouba, 1997). The three-dimensional context requires students to use formulas to compute surface areas and volumes of figures such as cylinders, prisms, pyramids, and cones. The State of Washington specifies in its Grade Level Expectations (2004) that students grades 9 and 10 "apply formulas to calculate measurements of right prisms or right circular cylinders" (p. 24) and "understand the relationship among characteristics of one-dimensional, two-dimensional, and three-dimensional figures" (p. 26). Strutchens (2001) states that "More often than not…students learn measurement through memorizing formulas rather than exploring the underlying concepts" (p. 402). To try to address the difficulties that students face in the three-dimensional context the researcher will implement activities designed to help increase students' understanding of three-dimensional figures and, as a result, effectively solve three-dimensional mathematics problems. The successful application of these activities should lead to higher levels of thinking among students and the activities will also serve as a resource to teachers of high school geometry.

## Background

Geometry, and especially spatial reasoning, is difficult for many high school mathematics students. In particular, Bako (2003) suggests that students "cannot see in 3-D" (p. 1). In addition, Strutchens (1997) uses findings from the 1992 NAEP to state that “as the geometric figures…became more complex…performance levels decreased” (p. 165).

Pierre van Hiele and Dina van Hiele-Geldof (husband and wife from the Netherlands) gave separate doctoral dissertations in 1957 (van Hiele, 1957; van Hiele-Geldof, 1957) at the University of Utrecht originating a theory that described how students learn geometry (Usiskin, 1982). The theory gained popularity in the United States through large studies in the early 1980’s in which the theory was investigated (Burger & Shaughnessy, 1986; Usiskin, 1982). Pierre van Hiele published *Structure and Insight* in 1986 further describing their theory (van Hiele, 1986). The van Hiele theory is comprised of five developmental levels which describe the geometric reasoning development of students who are learning geometry. The theory suggests a hierarchy of levels of understanding that students progress through as they effectively learn material. “It is very usual”, says van Hiele, “though always condemnable, to speak to pupils about concepts belonging to a level that they have not at all attained. This is the most important cause of bad results in the education of mathematics.” (p. 66). Students suffer frustration and discouragement when teachers instruct at levels for which they are not prepared, use unfamiliar language, or use a variety of problem-solving processes (Burger & Shaughnessy, 1986). Consequently, identifying the van Hiele level at which students, encountering three-dimensional problems, function is important. Gutierrez et al. (1991) created a method to relate three-dimensional spatial reasoning to the van Hiele theory. This method led to a model for assuming student levels of reasoning when problem solving. The model is crucial to measuring achievement in this project.

## Purpose of the study

The purpose of this project is to investigate the relation of the van Hiele theory to three-dimensional problem solving among $10^{th}$ grade geometry students in a south eastern Washington State high school. The researcher created a series of activities to supplement a geometry unit on three-dimensional surface area and volume. The activities were modeled after activities that will be discussed in the review of literature. The aim of the curriculum supplements was to increase students' abilities to determine the surface area and volume of prisms and pyramids. A secondary purpose of this study was to examine if the especially designed activities improved the students' levels of thinking with respect to the van Hiele levels of reasoning.

## Hypotheses

1. Students who participate in instructional activities that are based on the van Hiele theory will experience a statistically greater increase in van Hiele level of reasoning than those students receiving traditional instruction.
2. Students who participate in instructional activities that are based on the van Hiele theory will experience a statistically greater increase in ability to determine surface area than those students receiving traditional instruction.
3. Students who participate in instructional activities that are based on the van Hiele theory will experience a statistically greater increase in ability to determine volume than those students receiving traditional instruction.

## Rationale

Studies have shown that students tend to enter geometry at low van Hiele levels (Senk, 1989; Usiskin, 1982). The difficulties that the researcher has observed in his own classroom suggest that this is also true for his students. Mistretta (2000) and Fuys et al. (1988) used activities and instruction formed from the van Hiele theory to help students increase their levels of understanding of basic geometry concepts, thus improving their achievement. Consequently, one might conjecture that similar activities in a three-dimensional context might have similar effects. If students experience an increase in van Hiele level, they might also see an increase in achievement in three-dimensional geometry. Gutierrez et al. (1991) use a "flexible interpretation of the van Hiele theory" (p. 249) to present a model for the evaluation of the van Hiele levels of thinking in the context of three-dimensional Geometry, which the researcher proposes to modify for the project described herein.

## Limitations and Delimitations

This study is conducted under the quasi-experimental design, using control and treatment groups. However, each group is a convenience sample made up of a geometry class that the researcher teaches. Consequently, generalized results will have limited value. All subjects will take a spatial geometry test modeled after that of Gutierrez et al. (1991) before and after the treatment. Differences between the scores will be analyzed. An interval of two to three weeks will separate the pre/post test. Consequently, item retention may bias post test performance.

## Definition of Terms

1. Two-dimensional Euclideon geometry—geometry related to the Cartesian plane that involves points, lines, rays, and figures in the plane such as triangle, quadrilaterals, and other polygons.

2. Three-dimensional Euclidean geometry—geometry that places a third axis perpendicular to the Cartesion plane. Objects in this setting include cylinders, prisms, cones, pyramids, and spheres.
3. Van Hiele Level Theory—"A model of the development of geometric thinking that identified five differentiated levels of thinking, ordered so that the students moved sequentially from one level of thinking to the next as their capability increased" (Gutierrez et al, p. 237).
4. Net—A diagram of a hollow solid consisting of the plane shapes of the faces so arranged that the cut-out diagram could be folded to form the solid (Lawrie, 2000).
5. Traditional instruction as related to surface area and volume—Instruction solely rooted in the researcher's textbook curriculum, McDougal Littell Geometry 2001.

# CHAPTER 2

# REVIEW OF THE LITERATURE

## Introduction

The State of Washington specifies in its Grade Level Expectations (2004) that students grades 9 and 10 "apply formulas to calculate measurements of right prisms or right circular cylinders" (p. 24) and "understand the relationship among characteristics of one-dimensional, two-dimensional, and three-dimensional figures" (p. 26). The National Council of Teachers of Mathematics (NCTM) also has standards to this effect (NCTM, 2000). However, these skills are problematic for many geometry students.

Several studies have noted the difficulties that students face in learning geometry concepts such as proof (Senk, 1989; Usiskin 1982), area (Mistretta, 2000; Strutchens, 2001), measurement (Strutchens, 2001), and reasoning and relational understanding (Mistretta, 2000). Swafford et al. (1997) describe the achievement of exiting eighth graders on national and international assessments as "markedly low" (p. 467).

Usiskin (1982) points out that high school geometry in the United States is usually studied in a single year in which students are introduced to the major concepts of geometry (i.e. definitions, postulates, theorems, and proof) while assuming little prior content knowledge. Consequently, the abstract nature of this course poses much difficulty to many students. What van Hiele (1986) described when teaching his students may also be the experience of many teachers, "it always seemed as though I were speaking a different language" (p. 39).

The additional concepts which relate to three-dimensional geometry are often introduced late in the year with the properties of prisms, pyramids, and spheres being briefly introduced and their surface area and volume formulas derived and used to solve problems (Coxford et al., 1991). Condensed and hasty exposure is problematic for the following reasons. Firstly, Gutierrez (1992) suggested that students usually work in three contexts when they handle three dimensional objects; namely, 1) they manipulate actual physical objects, 2) they manipulate representations of the objects on a computer screen, or 3) they manipulate plane representations such as pictures or drawings on paper. Conversely, the traditional approach to instruction of three-dimensional geometry avoids any opportunities to interact with manipulatives, which would help students develop the three dimensional concepts and spatial reasoning. Furthermore, traditional instruction often restricts the learning experience to a single context, the textbook representations of the three-dimensional figures.  Coxford et al. (1991) suggest that this approach to teaching solids (i.e. through formulas and without manipulatives) should be modified to introduce the solids through the shapes of their faces, such as triangles and quadrilaterals. The resulting solids are pyramids, prisms, and tetrahedra. Coxford et al. (ibid.) further recommend that students explore the materials in different contexts in order to better visualize the 3-D shapes and generalize the constructive process. This approach, incidentally, assumes students begin at the entry level of the van Hiele model.

## Van Hiele Level Theory

Pierre van Hiele's (1986) level theory has been applied to explain why students have difficulty with high level cognitive processes required in geometry. Mistretta (2000) cites Lawrie and Pegg to describe the theory as a "developmental model of thought processes through which students progress as they learn geometry" (p. 366). Burger and

Shaughnessy (1986) and Usiskin (1982) provide a helpful reference in summarizing the five van Hiele levels as listed below:

Level 1 (Recognition): The student can learn names of figures and can recognize a shape as a whole. (Squares and rectangles seem to be different.)

Level 2 (Analysis): The student can identify properties of figures. (Rectangles have four right angles.)

Level 3 (Abstraction): The student can logically order figures and relationships, but can not operate within a mathematical system. (Simple deduction can be followed, but proof is not understood.)

Level 4 (Deduction): The student understands the significance of deduction and the roles of postulates, theorems, and proof. (Proofs can be written with understanding.)

Level 5 (Rigor): The student understands the necessity for rigor and is able to make abstract deductions. (Non-Euclidean geometry can be understood.)

Myers (2009) states that “the van Hiele theory is based on instructional techniques and not on age” (p. 24). Consequently, the van Hiele’s (1986) also specified phases of learning that teachers should integrate into lessons that teach geometry. Mistretta (2000) provides an outline of these five phases given below:

1. Information: Discussions are held where the teacher learns of the students’ prior knowledge and experience with the subject matter at hand.
2. Direct Orientation: The teacher provides activities that allow students to become more acquainted with the material being taught.

3. Explication: A transition between reliance on the teacher and students' self-reliance is made.
4. Free Orientation: The teacher is attentive to the inventive ability of the students. Tasks that can be approached in numerous ways are presented to the students.
5. Integration: The students summarize what was learned during the lesson.

It has been theorized (Usiskin, 1982) that students who experience difficulty in geometry are being taught at too high a van Hiele level, when they should be taught sequentially through each level. Van Hiele (1986) suggests that students are unable to advance without mastering each prior level. Pre and Post test scores from the van Hiele Geometry Test indicated to Usiskin (1982) that 40% of high school students remain at a junior high level (i.e. Level 1) of understanding at the end of a year-long geometry course. Consequently, students tend to leave the tenth grade with the same understanding as when they were in junior high. This is striking when contrasted with the observation that students need to demonstrate at least a Level 2 understanding to be successful in high school geometry (Mistretta, 2000; Senk, 1989).

## Three-Dimensional Context

As seen above, the van Hiele levels are predominately described by, and applied to, two-dimensional geometry concepts, such as lines, triangle congruence, and proof. However, Gutierrez et al. (1991) proposed a "flexible interpretation" (p. 249) of the level descriptors to suit three-dimensional geometry which they subsequently used to classify students in that context. Gutierrez (1992) states that, while the van Hiele levels are accurately identified for some geometrical topics, little is known about them in three-dimensional geometry. In fact, the difficulty that students face in the three-dimensional setting is a little understood topic. Studies have investigated the relationship between the van Hiele levels and geometry achievement in general (Fuys et al., 1988; Mistretta, 2000; Usiskin, 1982) and achievement in proof writing in particular (Senk, 1989). Mistretta (2000). Fuys et al. (1988) used activities and instruction formed from the van Hiele theory to assist students in their increase in level of understanding of basic geometry concepts and subsequent improvement in achievement. Studies have investigated the role of three-dimensional computer software to increasing the understanding of properties of three-dimensional figures (Bako, 2003). However, little has been done relating the van Hiele levels of thinking and three-dimensional geometry. Perhaps this is due to the fact that the van Hiele theory was developed to deal with students' abstract concept of proof. Alternatively, the context of three-dimensional geometry is often concrete and requires students to solve problems that may involve manipulatives, illustrations, and calculations. Consequently, Gutierrez et al. (1991) provide descriptors of the van Hiele levels suitable to the three-dimensional context. These are summarized below and were used in this study:

> <u>Level 1</u> (Recognition): Solids are judged by their appearance. The students consider three dimensional objects as a whole. They recognize and name solids and can distinguish a given solid from others on a visual basis.
>
> <u>Level 2</u> (Analysis): The students identify the components of solids (faces, edges, etc.) and the solids are bearers of their properties (parallelism, regularity, etc.). They are not able to logically relate the properties to each other, nor can they logically classify solids or families of solids.

Level 3 (Informal Deduction): The students are able to logically classify families of solids (classes of prisms or rounded solids, regular polyhedral, duality, etc.). They can give informal arguments for their deductions, and they can follow some formal proofs given by the teacher or the textbook, but they are only able to carry out simple inference by themselves.

Level 4 (Formal Deduction): The students understand the role of the different elements of an axiomatic system (axioms, definitions, undefined terms, and theorems). They can also perform formal proofs.

There is evidence that students have difficultly transitioning between two-dimensional "planar" geometry and the three-dimensional geometry of solids (Bako, 2003; Gutierrez, 1992). Consequently, the teaching of three-dimensional geometry requires that a teacher use models since students "cannot see in 3D" (Bako, 2003). Bako gives examples of such models as those made from paper, models from transparencies (plastic paper), assembled plastic pieces, models made of wood or plastic, and figures demonstrated on a computer screen. These examples can be categorized under Gutierrez's (1992) observation that there are three contexts in which students work when studying three-dimensional geometry: manipulation of physical objects, manipulation of three-dimensional representations on a computer screen, and reading or drawing plane representations on paper. Gutierrez (ibid) further points out that, while plane representations are the most frequent encountered in our world and while they supply the most complete information about the characteristics of the represented solids, they are nonetheless the most difficult to mentally manipulate. It follows that students should be taught to manipulate what they see most frequently in their lives. The National Council of

Teachers of Mathematics suggests that "solids should be introduced with the shapes that describe their faces" (p. 43). They further state that two major goals of "the early introduction of polyhedra" are to "improve the visualization and sketching skills of students" (p. 43). Consequently, it seems appropriate to help students bridge the gap between observing a three dimensional object, recognizing its properties, and ultimately, effectively representing those with paper and pencil.

## Instructional Implications of the van Hiele Theory

The van Hiele theory has student-related and teacher-related implications. As alluded to previously, and as stated by Lawrie et al. (2000), the van Hiele theory serves as a "hierarchy of growth" (p. 3-218) through which a student must progress in order to gain successful understanding. Van Hiele (1986) and Mistretta (2000) outline the five phases of learning that need to be integrated into geometry lessons as forwarded by van Hiele-Geldof (1984). Consequently, teachers of geometry have a responsibility to structure their lessons in a way that enables students to make the necessary transition from one level of reasoning to the next, especially since traditional geometry instruction is presented at level 3, deduction (van Hiele, 1986). This necessary progression is illustrated in the following table, adapted from Genz (2006):

Table 1: Van Heile's Model of Instruction

| | |
|---|---|
| **Abstraction**<br>(Level 3) | Student understands properties of geometric shapes, forms definitions, and understands necessary and sufficient properties. |
| **Learning Period 2** | Phases of Learning<br>Integration<br>Free Orientation<br>Explication<br>Guided Orientation<br>Information |
| **Analysis**<br>(Level 2) | Student reasons about geometric concepts by an informal analysis of component parts. |

| | |
|---|---|
| ↑ **Learning Period 1** | <u>Phases of Learning</u><br>Integration<br>Free Orientation<br>Explication<br>Guided Orientation<br>Information ↑ |
| **Recognition**<br>(Level 1) | Student reasons about basic geometric shapes by visual considerations. |

The theories of van Hiele (1986), van Hiele-Geldof (1984) provide a paradigm from which to base instructional methods, strategies, and materials. Insight into the thinking of students and learning phases can positively impact teaching. Swafford (1997) showed that elementary teachers who tended to not teach geometry because of lack of content knowledge were positively impacted through geometry instruction and instruction concerning students' cognition. She concluded that knowledge of these two factors affected what geometry was taught, how it was taught, and the professional characteristics that the teachers exhibited while teaching geometry. Van Hiele (1986) states that using principles of the theory alone would yield positive results regardless of the curriculum.

Research supports the researcher's intentions of implementing a curriculum modification based on the van Hiele theories of thinking and phases of learning. This modification will aim to bridge the gap between two-dimensional and three-dimensional geometry through activities based on the van Hiele levels of thinking. Isolated activities have been forwarded to help students understand different aspects of three-dimensional geometry (Bako, 2003; Coxford et al., 1991; Del Grande et al., 1993; Johston-Wilder 2005; Strutchens et al., 2001). Other activities have been shown to raise students' level of thinking (Fuys et al., 1988; Gutierrez, 1992; Mistretta, 2000). Therefore, a curriculum modification that unites these activities with the purpose of increasing students' van Hiele levels of thinking as well as understanding of three-dimensional geometry is justified. Specifically, the activities will target students' understanding of, and ability to, determine surface area and volume of right-triangular and right-rectangular prisms and right-triangular and right-square pyramids.

# CHAPTER 3

# METHODS

## Participants

The participants of this study consisted of a convenience sample of 51 9th-grade Geometry students from Pasco High School in Pasco, WA. The target population of this study is high school geometry students. Pasco High School has a student enrolment of approximately 2000 students, 67% of whom qualify for free or reduced-price meals. The ethnic makeup of Pasco High is approximately 75% Hispanic, 20% Caucasian, and 5% other. The participants of this study were divided between two classes currently being taught by the researcher. Both classes reflect the ethnicity of the high school as a whole. Additionally, both classes are taught in the morning with one occurring 1st period (7:50-8:43) and the other occurring 4th period (10:50-11:43) every day.

A quasi-experimental design was used wherein the 1st period class of 30 students was designated the treatment group and the 4th period class of 21 students was designated the control group. The dependent variable in question was the level of understanding demonstrated by students when solving surface area and volume problems. The treatment variable was the type of instruction used within each group. The control group received traditional instruction used by the researcher in the past which relied heavily on direct instruction and assigned bookwork. The treatment group received modified instruction which introduced a series of activities aimed at increasing students' van Hiele level of understanding.

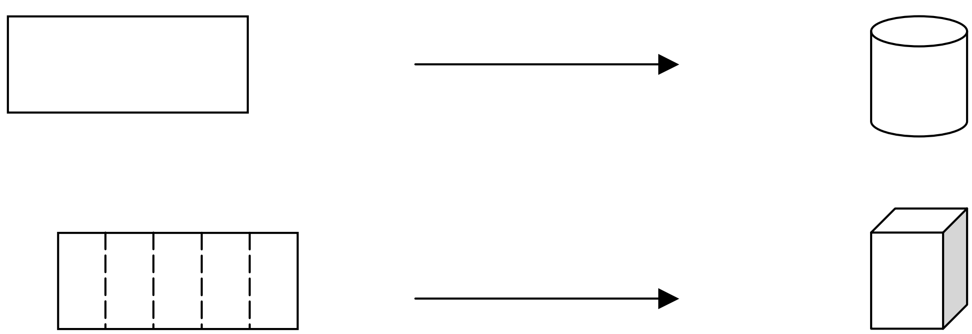
Description of Control and Treatment Instruction

The unit of instruction occurred over a period of two weeks. The treatment group engaged in three activities for approximately four days at the beginning of the unit while the control group received traditional instruction. The experience of the researcher teaching the final unit of the geometry curriculum, which involved surface area and volume, was not significantly different from that found in the literature. That is, the concepts were introduced late in the year without the use of manipulatives. Students were expected to learn the features of solids through textbook examples and problems and memorize the corresponding formulas which, were used to determine surface area and volume. Consequently, the result of the traditional approach was that students had not developed the vocabulary used to describe solids (e.g. faces, edges, base) and were often unable to identify the features of the solid that each formula referred to (slant height, area of the base, etc.). Furthermore, to compensate for the low level of understanding by students, teachers often include only assessment items that are familiar to the students from middle school, such as rectangular prisms and cylinders, as opposed to more complex solids like hexagonal or octagonal prisms and pyramids.

To address the deficiencies of traditionally teaching of three-dimensional geometry, a curriculum modification consisting of three activities was given to a control group of students. These activities introduced the unit through manipulatives and required students to compare and contrast solids and to identifying key features of solids using the appropriate vocabulary. The following three activities were incorporated into existing Geometry curriculum for the purpose of enhancing the van Hiele level of thinking demonstrated by the researcher's students. Mistretta (2000) refers to Fuys et al. (1988) in stating that “teaching techniques advocated by the van Hiele’s allow students to learn geometry by means of hands-on activities. The students utilize problem-solving strategies that, when combined with concrete experiences, yield higher order thinking skills” (p. 368). Mistretta (ibid) also quoted Pierre van Hiele as saying that “geometry begins with play” (p. 440). Thus, the aim of these activities is provide a fun and interesting means to guide the students through the vocabulary and properties of geometric solids in order to assist them in attaining higher order thinking skills with regard to solving surface area and volume problems.

The first activity, taken from Koester (2003), highlighted the faces of a solid by requiring students to form shapes of geometric solids using solid pieces of paper and nets. This is done through rolling up pieces of paper (producing cones and cylinders) and folding pieces of paper (producing prisms). The transition from rectangles and circles (2-D) to cylinders, prisms, and pyramids (3-D) is illustrated in the following table:

**Rectangles to cylinders and prisms**



**Circles to cones and pyramids**

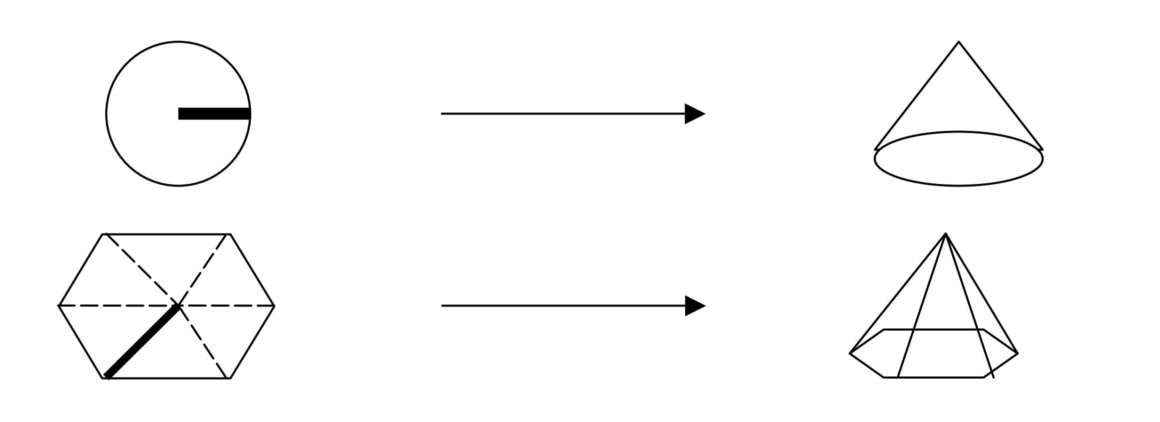


*Figure 1*. Transition from 2-D to 3-D

Once students finished creating the paper models, they were asked to classify their solids as polyhedra or non-polyhedra. At this point vocabulary such as faces, edges, vertices, and bases was introduced as students made observations. This activity fosters Level 1 reasoning by asking students to consider the objects as a whole, recognize and name them, and distinguish each solid visually from the others.

An extension to the first activity was to ask students to further classify their solids as being a prism, pyramid, cylinder, or cone. Students used a large piece of butcher paper to create the following grid upon which they placed each solid as seen below in Figure 2.

| | |
|---|---|
| Prism | Cone |
| Pyramid | Cylinder |

*Figure 2.* Scheme for classifying solids

The second activity also used by Koester (ibid.) and referred to by Mistretta (2000) asked students to discover the relationship between the edges, vertices, and faces of the polyhedra and to determine Euler's Formula. Figure 3 below shows a chart that students used to guide their exploration and record their information.

| Name of shape | Number of Sides for the base | Number of faces | Number of Vertices | Number of Edges |
|---|---|---|---|---|
| Triangular Prism | | | | |
| Rectangular Prism | | | | |
| Pentagonal Prism | | | | |
| Hexagonal Prism | | | | |
| n-gonal Prism | | | | |

*Figure 3.* Chart to investigate Euler's Formula

Mistretta (ibid.) states that students who can use this activity to analyze the concluding data and discern the pattern of Euler's formula (number of faces plus number of vertices minus 2 equals number of edges) are thinking at van Hiele level 2.

The third activity required students to draw three dimensional shapes. Bako (2003) states that "the axonometric projection is the most used representation method" and that "almost every textbook uses this method" (p. 3). The axonometric projection of a three dimensional solid is the form of illustration where parallel lines stay parallel. It is best understood in contrast to the central projection which deforms sizes as illustrated below (see Figure 4). Furthermore, Bako states that “it is not enough to copy drawings, students need to interpret what they see.” (p. 8). Consequently, students practiced creating axonometric drawings of a variety of solids in order to help them understand the three-dimensional figures that they observe in their textbooks. Students also practiced labelling these drawings and identifying their properties.

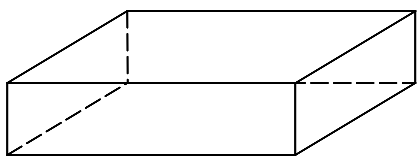
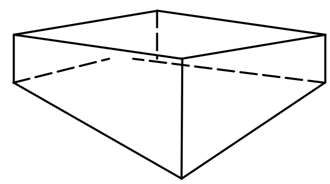
| **Axonometric Projection** | **Central Projection** |
| --- | --- |
|  |  |

*Figure 4.* Axometric Projection vs. Central Projection

Students began solving surface area and volume problems once they have had sufficient practice drawing different solids and identifying properties from their drawings.

Assessment of van Hiele Levels Instrument

The researcher attempted to model the work done by Gutierrez et al. (1991) by creating an assessment, dubbed the Surface Area and Volume Assessment, or SAAVA (in Appendix A), which was used to measure the van Hiele level of students’ thinking as they solve surface area and volume problems. SAAVA scores were used to assign each student a particular van Hiele level as indicated by student responses within the assessment. Differences between the pre and post SAAVA scores indicated any differences in achievement by the students involved in the study. In harmony with his advisor, the researcher created questions which, modeled after Gutierrez’s work, provided an accurate reflection of the van Hiele levels of thinking in the context of three-dimensional problem solving.

Test-retest reliability of the SAAVA was determined by administering a pilot test to a class of 30 9$^{th}$ grade Algebra students and comparing their pre- and post- test responses. The pre-test was administered during the week prior to the students' two-week Christmas break. The post-test was administered upon their return. Responses were checked and closely scrutinized for indications that the pre-test influenced the post-test. Student scores showed a correlation of .77 which indicated a reasonable level of reliability.

The researcher is confident that the assessment results will prove valid due to, a) the work done by Gutierrez et al. (1991) as a basis of the SAAVA, b) the involvement of the researcher's advisor in creating the assessment and c) the length of time between the pre- and post- tests. The nature of the Christmas break between the testing periods is assumed to have significantly reduced any testing threat to validity.

## SAAVA Scoring Guidelines

The questions within the SAAVA were designed to evaluate the van Hiele level of students' thinking in three-dimensional geometry. The assessment targeted only van Hiele levels 1 and 2 since the research indicates that Geometry students enter at low levels of understanding and need at least Level 2 understanding to be successful in high school geometry (Mistretta, 2000; Senk, 1989). Of the eighteen items on the assessment, thirteen were designed to assess Level 1 thinking and five were designed to assess Level 2 thinking. More items targeted Level 1 since acquisition of this level is required to demonstrate Level 2.

Webb's Depth of Knowledge (DOK) was referred to when scrutinizing SAAVA items and assessing students' results. Myers (2009) provides a comparison between the van Hiele levels, Bloom's taxonomy, and Webb's DOK. Williams (2011) describes DOK as "Measure[ing] the degree to which the knowledge elicited from students on assessments is as complex as what students are expected to know and do." Consequently, the level characteristics from van Hiele and Webb were combined to ensure that the intent of the van Hiele level of student achievement matched the SAAVA assessment items. The result is the SAAVA scoring guidelines in Appendix B.

When scoring student responses, Gutierrez et al. (1991) used an approach that identified the level of thinking demonstrated in the response and the type of response. The level of thinking was primarily determined by the level reflected by the problem itself. The type of response was a measure of how fully the student demonstrated that level of thinking as revealed by the characteristics of the van Hiele levels and Webb's DOK. Thus, every item on the SAAVA was rated by the percent to which students demonstrated the required level of thinking (0-100). Final scores were an average of all of the items at each particular level, whether Level 1 or Level 2. The rubric used to score the items is in Appendix B.

Table 2 below is an example of a students' assessment results. Each column of the table represents a SAAVA item. The horizontal rows represent each level of thinking. The final column shows that the student demonstrated near complete Level 1 thinking at 92.7% in addition to showing 62% Level 2 ability.

Table 2
*Student Scoresheet Example*

| Post | | | | | | | | | Items | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Level | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2a | 2b | 2c | 3a | 3b | 4 | 5a | 5b | 5c | 5d | Average |
| 1 | 100 | 100 | 100 | 50 | 100 | 100 | 100 | 100 | 100 | 75 | – | 80 | – | – | 100 | 100 | – | – | 92.7 |
| 2 | – | – | – | – | – | – | – | – | – | – | 100 | – | 80 | 50 | – | – | 80 | 0 | 62 |
| 3 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 4 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |

## Process for Analysis of Data

The researcher was solely responsible for evaluating the pre- and post- SAAVA scores for each of his students. The performance characteristics of each van Hiele level of reasoning in addition to Webb's DOK descriptors provided objective parameters with which to scrutinize student results. The following steps were followed once the students' scores were obtained.

Gains in levels of reasoning were determined through the difference between the pre- and post- SAAVA scores. Descriptive statistics were used to construct side-by-side boxplots for the control and treatment groups and the results compared. This initial comparison was used as an initial test of the researcher's first hypothesis. A similar analysis was performed using the student scores resulting from the Surface Area and Volume items from the SAAVA in order to test hypotheses 2 and 3 respectively. An independent samples *t*-test was used to compare the average differences when appropriate. The Mann-Whitney *U* test was used to compare means when the normal distribution of scores for the *t*-test were not satisfied.

ANCOVA was attempted between the pre- and post- test results in order to determine statistical significance and interaction between the groups. The students' SAAVA pre-test results was used as the covariate.

# CHAPTER 4

## Data Analysis

The Surface Area and Volume Assessment, or SAAVA (see Appendix A) is a tool created by the researcher to assign students a van Hiele level based on a 100-point scale. In particular, the SAAVA attempts to determine the van Hiele level that students demonstrate when solving surface area and volume problems. The results from the SAAVA pre-test and post-test revealed two measurements for each student; namely, average ratings of Level 1 and Level 2 thinking ability. The results of each level were analyzed separately using SPSS statistical software and Excel data analysis tools. Higher van Hiele levels of reasoning (levels 3 or 4) were not assessed by the SAAVA since the researcher's interest lay in the level at which students enter geometry, which is Level 1 or below according to the research.

### Results Hypothesis 1

#### *Analysis of Level 1 reasoning*

The first hypothesis stated that students who participate in instructional activities that are based on the van Hiele theory will experience a statistically greater increase in van Hiele level than those students not receiving the supplementary activities. A one-way ANCOVA was conducted in which the independent variable was the class in which the students were enrolled (treatment or control) and the dependent variable was the degree of Level 1 thinking (0-100) demonstrated on the SAAVA post-test. The covariate for this test was students' SAAVA pre-test scores. A necessary condition to the use of ANCOVA is that the slopes of the regression lines modeling the post-test scores against the pre-test scores not be significantly different. The slopes of the regression lines for the control and treatment groups were 0.689 and 2.991 respectively (see Figure 5). Thus, the homogeneity of slopes condition was not met indicating interaction between the two independent variables. However, closer inspection of Figure 5 revealed three outliers: (1.5, 11.2), (7.7, 33.8), and (52.7, 98.1). This was confirmed using SPSS software and the 1.5*IQR rule. Consequently, these points were removed and a one-way ANCOVA repeated.

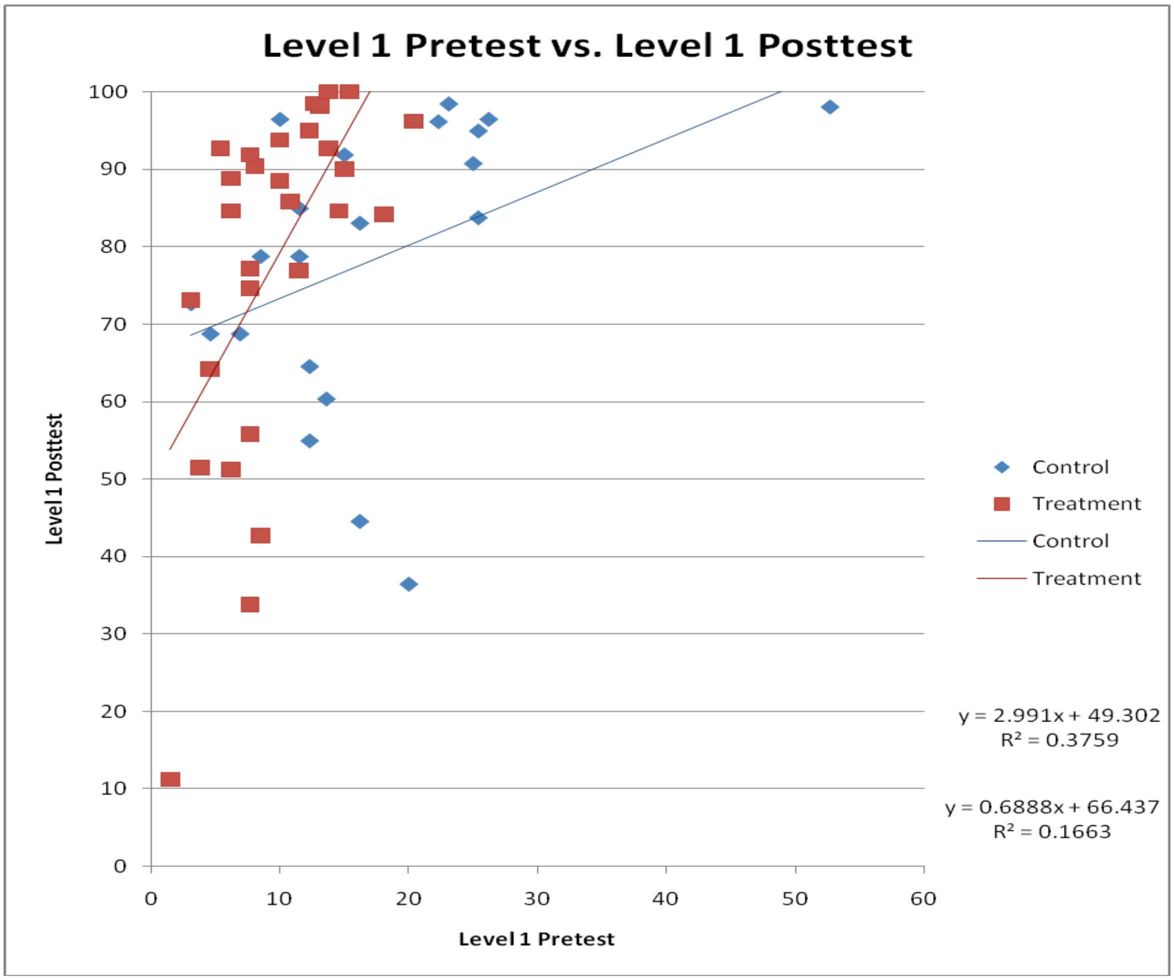



*Figure 5*. Graph showing the homogeneity of slopes test. Graph shows the correlation between the Level 1 thinking ability of the treatment and control groups between the subjects' pre- and post-test.

The secondary analysis of the homogeneity-of-slopes assumption (see Table 3) indicated that the relationship between the covariate (Level 1 pre-test scores) and the dependent variable (Level 1 post-test scores) did not differ significantly as a function of the independent variable (class treatment). Furthermore, the ANCOVA was significant, $F(1,45) = 5.95$, $MSE = 242.09$, $p < .02$.

Table 3
*Tests of Between-Subjects Effects: Level 1*

Dependent Variable:Level1Post

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
|---|---|---|---|---|---|---|
| Corrected Model | 2838.279[a] | 2 | 1419.139 | 5.862 | .005 | .207 |
| Intercept | 31281.161 | 1 | 31281.161 | 129.214 | .000 | .742 |
| Class | 1440.171 | 1 | 1440.171 | 5.949 | .019 | .117 |
| Level1Pre | 2472.972 | 1 | 2472.972 | 10.215 | .003 | .185 |
| Error | 10894.000 | 45 | 242.089 | | | |
| Total | 325396.380 | 48 | | | | |
| Corrected Total | 13732.279 | 47 | | | | |

a. R Squared = .207 (Adjusted R Squared = .171)

Level 1 results were computed (see Table 4) and analyzed, with the following observations.

Table 4
*Descriptive Statistics: Level 1*

| | | Mean (%) | Std. Deviation | N |
|---|---|---|---|---|
| Pre-Test | Control | 15.46 | 1.63 | 20 |
| | Treatment | 10.27 | 0.83 | 28 |
| Post-Test | Control | 77.32 | 4.05 | 20 |
| | Treatment | 82.91 | 3.07 | 28 |
| Difference | Control | 61.86 | 3.81 | 20 |
| | Treatment | 72.64 | 2.70 | 28 |

Firstly, it is evident that both groups of students entered the instructional unit with a low level of measurement ability, thus confirming the results from the literature review. Furthermore, a 95% confidence interval for the average entry-level ability of the students indicates that students entered the unit with an average Level 1 acquisition between 10.5% and 14.2%. Thus, students on average brought almost no detectable prior knowledge of surface area and volume into the instructional unit.

A second observation from the pre-test results, as seen in Figure 6, reveals that the control group entered the instructional unit with a higher level of reasoning ability as compared to the treatment group. An independent samples *t*-test was performed between the two groups showing a statistically significant difference between the two groups of students, thus confirming this observation, $t(46) = 3.08$, $p = .004$.

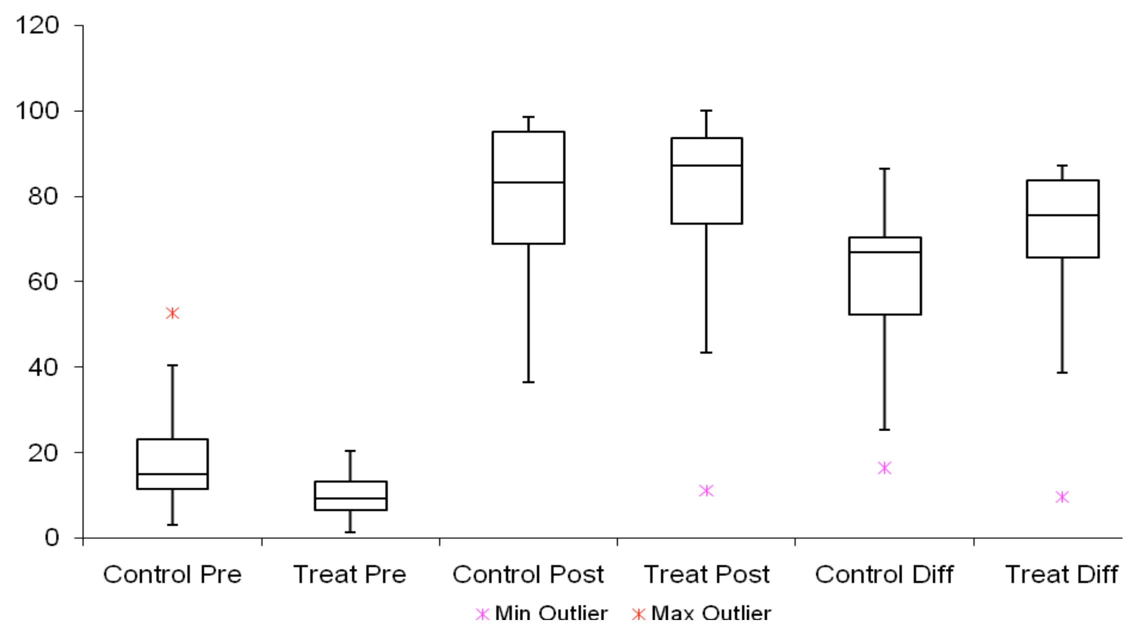



*Figure 6*. Boxplots of the Level 1 pre-test, post-test, and difference for the control and treatment groups' Level 1 reasoning ability.

A third observation from Table 3 is that Level 1 achievement gains, 61% and 73% for the control and treatment groups respectively, are quite different. Boxplots of this same data help to illustrate more clearly that the treatment group achieved a greater difference between the pre- and post-tests. Side by side boxplots are provided in Figure 6.

The researcher's use of an independent samples *t*-test to evaluate the differences in Level 1 reasoning was questionable due to the skewed distribution of the values. Consequently, a Mann-Whitney *U* test was conducted to evaluate the hypothesis that the treatment group's Level 1 achievement would be higher, on the average, than the control group. The results of the test was significant, with $z = -2.541$, $p < .05$. An independent samples *t*-test confirmed this result, $t(36.40) = -2.311$, $p = .016$, assuming a one-tailed *t*-test.

As a result of the above analyses, Hypotheses 1 was supported for Level 1 reasoning indicating that students who participated in instructional activities that were based on the van Hiele theory experienced a statistically greater increase in van Hiele Level 1 than those students who did not receive the supplementary activities.

*Analysis of Level 2 reasoning*

The preliminary analysis of the homogeneity-of-slopes assumption (see Table 5) for Level 2 reasoning indicated that the relationship between the covariate (Level 2 pre-test scores) and the dependent variable (Level 2 post-test scores) did not differ significantly as a function of the independent variable (class treatment). Furthermore, the ANCOVA was significant, $F(1,45) = 7.26$, $MSE = 493.08$, $p < .05$. Descriptive statistics of the Level 2 results (see Table 6) were computed and analyzed, with the following observations.

Table 5
*Tests of Between-Subjects Effects: Level 2*
Dependent Variable:Level2Post

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared |
|---|---|---|---|---|---|---|
| Corrected Model | 5701.735[a] | 2 | 2850.867 | 5.782 | .006 | .204 |
| Intercept | 51486.939 | 1 | 51486.939 | 104.418 | .000 | .699 |
| Class | 3580.224 | 1 | 3580.224 | 7.261 | .010 | .139 |
| Level2Pre | 3491.420 | 1 | 3491.420 | 7.081 | .011 | .136 |
| Error | 22188.745 | 45 | 493.083 | | | |
| Total | 113489.000 | 48 | | | | |
| Corrected Total | 27890.479 | 47 | | | | |

a. R Squared = .204 (Adjusted R Squared = .169)

Firstly, the pre-test results indicate that the control group entered the instructional unit with a nearly statistically significant higher Level 2 reasoning ability, $t(32.129) = 1.586$, $p = .122$. However, despite the apparent advanced standing of the control group, the treatment group achieved greater gains at the end of the unit, as indicated by the difference figures in Table 6.

Table 6
*Descriptive Statistics: Level 2*

| | | Mean (%) | Std. Deviation | N |
|---|---|---|---|---|
| Pre-Test | Control | 5.75 | 2.11 | 20 |
| | Treatment | 1.86 | 1.26 | 28 |
| Post-Test | Control | 34.20 | 5.03 | 20 |
| | Treatment | 47.96 | 4.61 | 28 |
| Difference | Control | 28.45 | 4.42 | 20 |
| | Treatment | 46.11 | 4.42 | 28 |

Additionally, Figure 7 contains side-by-side boxplots which illustrate this comparison. An independent samples *t*-test evaluating the average differences between the control and treatment groups Level 2 reasoning ability indicated that the gains were statistically significant, $t(46) = -2.744$, $p = .005$, assuming a one-tailed *t*-test. Consequently, Hypothesis 1 was supported for Level 2 reasoning.

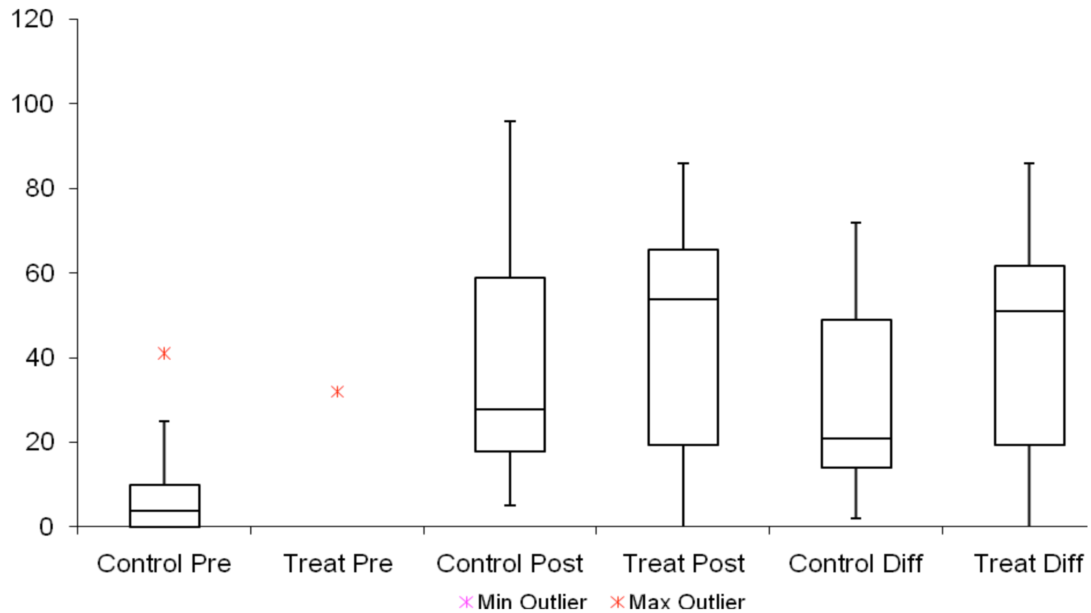



*Figure 7*. Boxplots of the Level 2 pre-test, post-test, and difference results for the control and treatment groups.

*Relationship between Level 1 and Level 2 reasoning*

Although not part of the researcher's original hypotheses, a comparison between Level 1 and Level 2 reasoning suggests a threshold in Level 1 reasoning where Level 2 acquisition begins to occur. Level 1 achievement is marked largely by recall of facts and definitions, or by solving single-step problems, and Level 2 achievement is demonstrated through solving multi-step problems. Consequently, it follows that students should demonstrate increasing Level 2 ability as they master Level 1 skills. Figure 8 shows a scatter plot of students' Level 1 and Level 2 post-test results, indicating that the two variables are linearly related such that Level 2 acquisition increases as Level 1 acquisition increases.

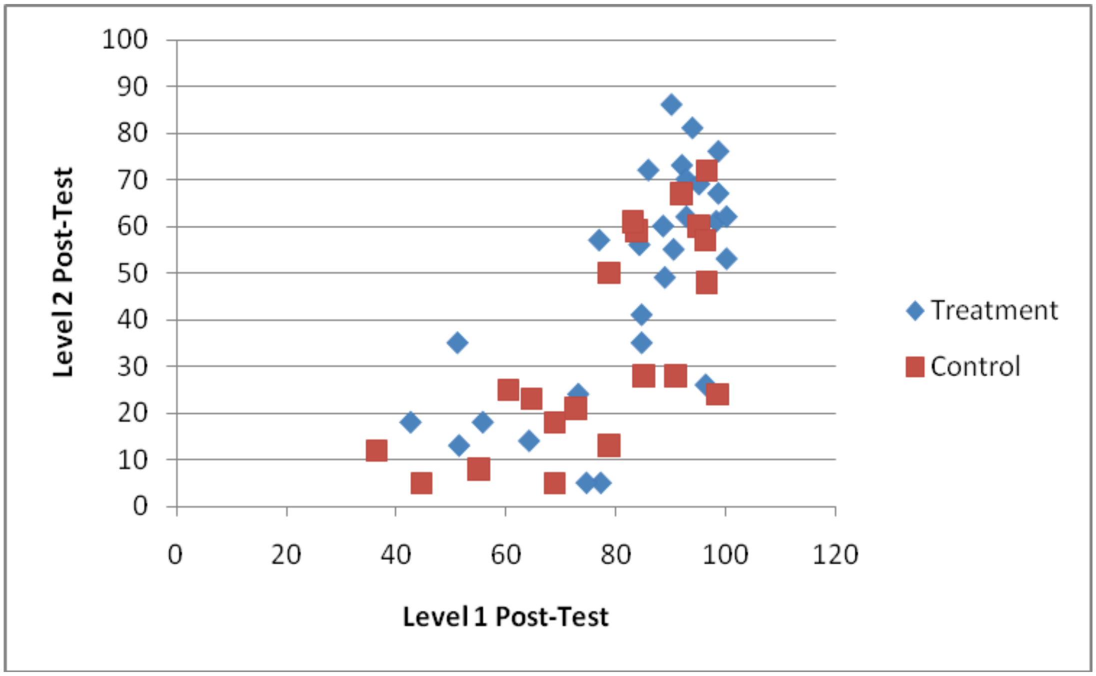



*Figure 8*. Scatter plot indicating Level 1 reasoning ability as a predictor of Level 2 acquisition.

A

re

ec

*Level 2 Posttest = 1.016 Level 1 Posttest – 39.634*

The correlation between the Level 1 Post-test and Level 2 Post-test was .734. Thus, approximately 51% of the variance in Level 2 acquisition was accounted for by its linear relationship with the Level 1 post-test results. Initially, this observation appears to contradict the van Hieles' hypothesis that a student must fully acquire a particular Level of reasoning before moving on to subsequent levels. The graph suggests that students can begin showing Level 2 acquisition after demonstrating only 40% of Level 1.

Further consideration of Figure 8, however, also confirms the findings of van Hiele when one recognizes the two clusters present in the graph. Van Hiele suggested that students must fully acquire a particular level of reasoning before progressing to a subsequent level. The clusters highlighted in Figure 9 suggest that students with greater Level 1 acquistion (> 80%) are able to show development in Level 2 reasoning (> 30%). Consequently, this supports that the treatment also had a significant effect on Level 2 acquisition.

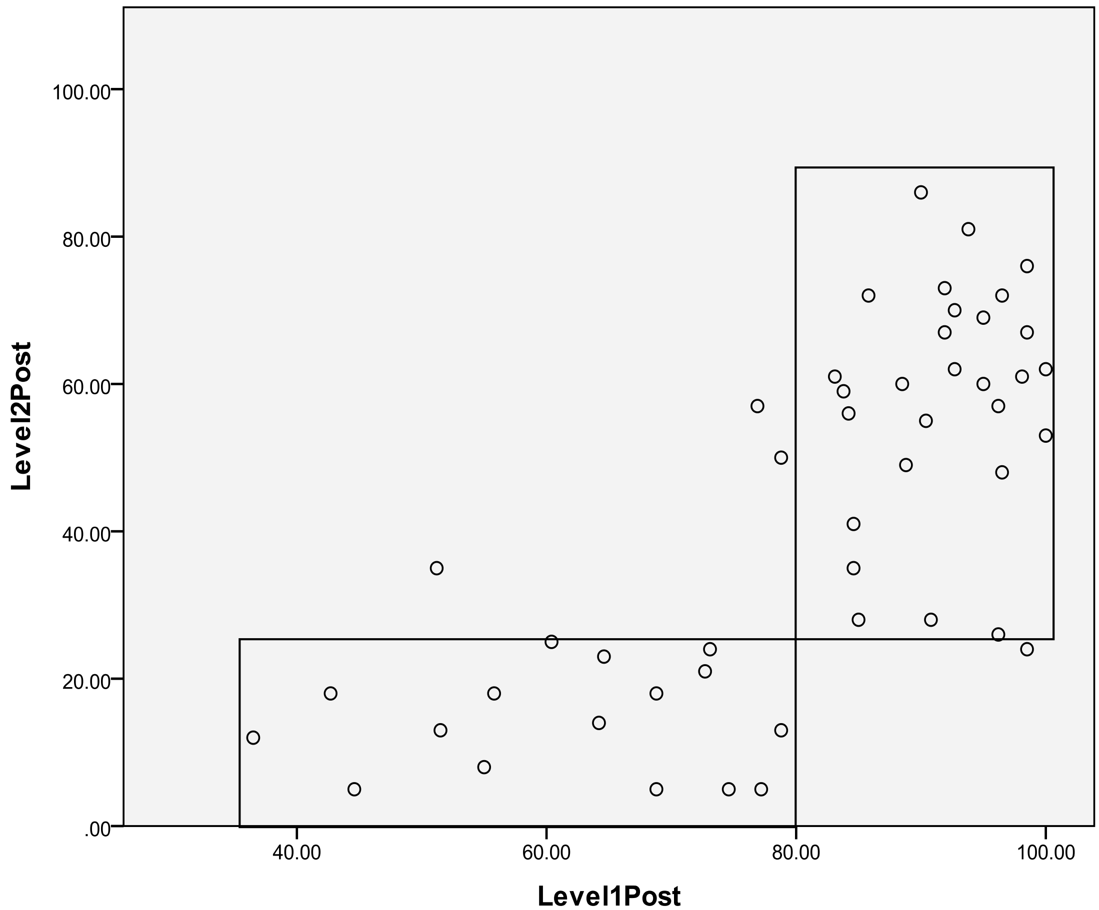



*Figure 9*. Clusters from a scatterplot of Level 1 vs. Level 2 post-test results suggest a minimum acquistion of Level 1 reasoning required for substantial Level 2 acquisition.

Results Hypotheses 2

The second hypothesis stated that students who participate in instructional activities that are based on the van Hiele theory will experience a statistically greater increase in ability to determine surface area. An observation of Figure 10 shows side by side boxplots of the disaggregated SAAVA scores that relate only to determining surface area.

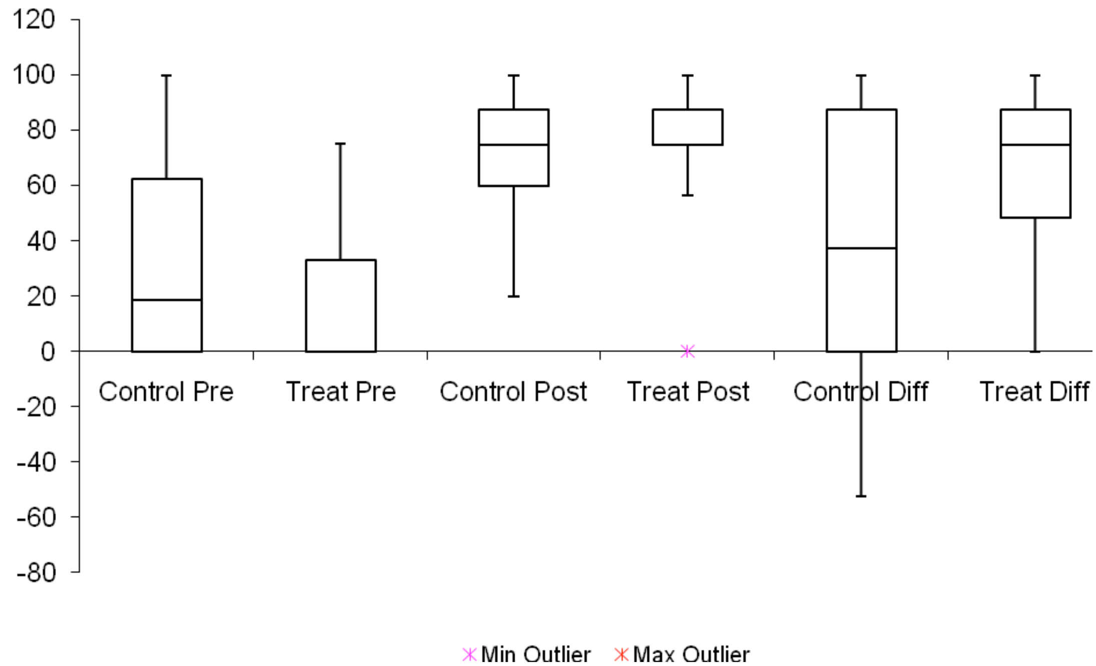



*Figure 10.* Boxplots of the disaggregated Surface Area pre-test, post-test, and difference results (%) for the control and treatment groups.

The symmetry of the difference in pre- and post-test scores suggests that an independent samples *t*-test is appropriate to compare the results of the two groups. This test was conducted to evaluate the hypothesis that the treatment group would score higher, on the average, than the control group when determining surface area with the following results; $t(29.385) = -2.453$, $p = .010$, assuming a one-tailed *t*-test. Consequently, Hypothesis 2 was supported. Descriptive statistics of the surface area results are contained in Table 7.

Table 7
*Descriptive Statistics: Determining Surface Area*

| | | Mean (%) | Std. Deviation | N |
|---|---|---|---|---|
| Pre-Test | Control | 33.30 | 7.88 | 21 |
| | Treatment | 13.17 | 3.79 | 30 |
| Post-Test | Control | 71.59 | 4.78 | 21 |
| | Treatment | 79.08 | 4.23 | 30 |
| Difference | Control | 38.30 | 10.28 | 21 |
| | Treatment | 65.92 | 4.60 | 30 |

Results Hypothesis 3

The third hypothesis stated that students who participate in instructional activities that are based on the van Hiele theory will experience a statistically greater increase in the ability to determine volume. Figure 11 shows side by side boxplots of the results from the students' performance on volume questions.


*Figure 11.* Boxplots of the disaggregated Volume pre-test, post-test, and difference results (%) for the control and treatment groups.

The results in Figure 11 are interesting to observe for the following reasons. Firstly, the 50% of the control group demonstrated almost 75% accuracy in measuring volume versus 50% of the control group scoring zero. This is yet another example of the control group's higher achievement entering the unit. However, it is also interesting to observe in Figures 11 and 12 that the control group alone experienced a negative difference between the pre- and post-test results.

Use of an independent samples *t*-test to evaluate the differences in determining volume was questionable due to the skewed distribution of the values. Consequently, a Mann-Whitney *U* test was conducted to evaluate the hypothesis that the treatment group would score higher, on the average, than the control group. The results of the test showed significance, with $z = -1.072$, $p < .05$. Incidentally, an independent samples *t*-test confirmed this result with, $t(36.255) = -1.735$, $p = .045$, assuming a one-tailed *t*-test. Consequently, Hypothesis 3 was supported. Table 8 contains the descriptive statistics of the disaggregated volume results.

Table 8
*Descriptive Statistics: Determining Volume*

| | | Mean (%) | Std. Deviation | N |
|---|---|---|---|---|
| Pre-Test | Control | 52.60 | 8.65 | 21 |
| | Treatment | 26.33 | 6.26 | 30 |
| Post-Test | Control | 83.64 | 4.14 | 21 |
| | Treatment | 78.75 | 4.79 | 30 |
| Difference | Control | 31.02 | 10.49 | 21 |
| | Treatment | 52.42 | 6.48 | 30 |

## Chapter 5

## DISCUSSION

### Summary

The purpose of this study was twofold. First the researcher wanted to investigate the level of thinking of students in Geometry in solving three-dimensional problems. Secondly, the researcher wanted to investigate the application of the van Hiele theory to measurement of Surface Area and Volume. The van Hiele level theory served as the basis for the investigation. The researcher's intent was to supplement the traditional geometry curriculum with a series research-based activities aimed at increasing the level of thinking among his students. A convenience sample of two geometry classes taught by the researcher allowed the researcher to compare the results of traditional instruction with those of the curriculum supplement. Statistical analysis of the results indicated that the effect of manipulative-based instruction of three-dimensional solids on students' level of reasoning and ability to determine surface area and volume was significant.

### Conclusions

Analysis of the results from the Surface Area and Volume Assessment (SAAVA) revealed the gains in reasoning ability achieved by the treatment group to be significantly greater than those of the control group. This included gains in Level 1 and Level 2 reasoning ability and gains in students' ability to determine Surface Area and Volume.

Results from the SAAVA also confirm the findings of prior research that students come into Geometry at significantly low levels of reasoning and are likely to remain below the desired Level 2 understanding by the end of their course in Geometry. However, it also appears that as students acquire more Level 1 reasoning ability, the more likely they will attain Level 2. Furthermore, a comparison between Level 1 and Level 2 post-test scores indicates that a Level 1 acquisition of approximately 80% is the minimum needed for significant Level 2 growth. This level of performance is described by the researcher's scoring rubric (see Appendix B) as "being able to provide correct answers that reflect a given level of reasoning but that are incomplete or insufficiently justified." In other words, students are capable of reaching the correct answer without sufficient explanation or justification. Students of this type can be expected to show growth at Level 2 while perhaps improving their ability to explain their work or justify their reasoning.

Negative post-test scores in volume and surface area were observed exclusively by the control group. The researcher suspects this was the result of students attempting to use the surface area and volume formulas introduced in the unit without sufficient Level 1 understanding to use those formulas. In other words, students used formulas in the post-test which they didn't understand. The researcher also suspects that this misunderstanding was largely the result of not being able to distinguish between the features of the solids that were represented in the formulas. For example, a student who attempts to use $V=Bh$ when determining the volume of a right prism may not realize that $B$ represents the area of the base. The area of the base of a prism is determined according to the shape of the base, whether square, pentagonal, hexagonal, etc. Students often use the faces of a prism, which are rectangular and easier to compute, in place of the base.

Finally, instruction that intentionally addresses students' phases of learning with respect to the van Hiele levels is seen, if subtly, to have a positive impact on students' development of Level 1 reasoning ability and beyond.

## Recommendations

The researcher intends to further his attempts to understand student thinking and how to inspire higher levels of reasoning by adapting his instructional methods to account for pre-assessment of student understanding and subsequent following of the van Hiele phases of learning. The process of pre-assessing the level of thinking of students and introducing research-based activities is a valuable exercise, which should be encouraged among educators. Creating the SAAVA became invaluable toward acquiring an in-depth understanding of the characteristics of levels of thinking and assessing them. Educators would do well to consider the characteristics of the levels of thinking presented here and the consequences of developing them among their students. This would require teachers to understand the levels well enough to identify them among their students, if only in an informal way, though an objective approach is preferred. Teachers should also consider scrutinizing their traditional instructional methods and assessments with respect to the van Hiele levels to ensure that they are not presenting material above what students are capable of mastering.

The results of this project reveal the importance of teaching students at the level of reasoning for which they are prepared. The research sends a clear message that geometry students tend to receive instruction that is above their thinking ability. Consequently, there is strong encouragement for educators to deliver instruction in a sequential way that begins at lower level thinking skills and progresses to higher levels.

Recommendations for future studies include looking further at the progression of the levels of thinking demonstrated by students in the three-dimensional context. Are there other hindrances to Level 2, 3, or 4 acquisition besides not mastering each prior level? Is there a threshold at a prior level (e.g. 80% proficiency) before a student can demonstrate proficiency at subsequent levels?

**Bibliography**

Bako, M. (2003). *Different projecting methods in teaching spatial geometry.* Retrieved July 4, 2007 from http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG7/TG7_Bako_cerme3.pdf.

Burger, W. F., Shaughnessy, J. M. (1986). Characterizing the van Hiele Levels of Development in Geometry. *Journal for Research in Mathematics Education*, vol. 17(1), 31-48.

Coxford, A.F., Burks, L, Giamati, C., Jonik, J. (1991). Geometry from multiple perspectives: Solids: Expanded perspectives. In Christian Hirsch (Series Ed.), *Curriculum and evaluation standards for school mathematics addenda series: Grades 9-12. Geometry from multiple perspectives* (pp. 43-46). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Del Grande, J. J., Morrow, L., Clements, D., Firkins, J., Joyner, J. (1993). Geometry and spatial sense: Chapter 3. In Miriam Leiva (Series Ed.), *Curriculum and evaluation standards for school mathematics addenda series: Grades K-6. Geometry and spatial sense* (pp. 25-29). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Fuys, D., Geddes, D., Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education. Monograph,* vol. 3.

Genz, R. (2006). Determining high school geometry students' geometric understanding using van Hiele levels: Is there a difference between standards-based curriculum students and nonstandards-based curriculum students? Retrieved August 23, 2010. http://contentdm.lib.byu.edu/ETD/image/etd1373.pdf.

Gutierrez, A. (1992). Exploring the links between van Hiele levels and 3-dimensional geometry. *Structural Topology*, 18, pp. 31-48.

Gutierrez, A. Jaime, A., Fortuny, J. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education,* 22(3), 237-251.

Johnston-Wilder, S., Mason, J. (2005). Block 2. In *Developing thinking in geometry* (pp. 98-101). London: Paul Chapman Publishing.

Kenney, P.A., & Kouba, V.L. (1997). What do students know about geometry? In P.A. Kenney & E.A. Silver, Eds., *Results from the sixth mathematics assessment of the National Assessment of Educational Progress*, (pp. 1-15). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Koester, B. (2003). Prisms and Pyramids: Constructing Three-Dimensional Models to Build Understanding. *Teaching Children Mathematics*, 9(8), pp. 436-442.

Lawrie, C., Pegg, J. Gutierrez, A. (2000). Coding the nature of thinking displayed in responses on nets of solids. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the conference of the international group for the psychology of mathematics education,* vol. 3.

Office of Superintendent of Public Instruction. (2004). *Mathematics: K-10 Grade level expectations: A new level of specificity*. Olympia, WA: Office of Superintendent of Public Instruction.

Mistretta, R. (2000). Enhancing geometric reasoning. *Adolescence,* 35(138), 365-379.

Myers, R.Y. (2009). The effects of the Use of Technology In Mathematics Instruction on Student Achievement. *FIU Electronic Theses and Dissertations*. Retrieved February 1, 2010. http://digitalcommons.fiu.edu/etd/136.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc.

Senk, S. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education,* 20(3), 309-321.

Strutchens, M.E., Harris, K.A., Martin, W,G. (2001). Assessing geometric and measurement understanding using manipulatives. *Mathematics Teaching in the Middle School,* 6(7), 402-405.

Strutchens, M.E. & Blume, G.W. (1997). What do students know about geometry? In P.A. Kenney & E.A. Silver, Eds., *Results from the sixth mathematics assessment of the National Assessment of Educational Progress*, (pp. 165-193). Reston, VA: The National Council of Teachers of Mathematics.

Swafford, J.O., Graham, A.J., Thornton, C.A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education,* 28(4), 467-483.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (final report of the Cognitive Development and Achievement in Secondary School Geometry Project). Chicago: University of Chicago, Department of Education. (ERIC Document Reproduction Service No. ED 220 288).

Van Hiele, P.M. (1957). *De problematiek van het inzicht gedomonstreed van het inzicht von schoolkindren in meetkundeleerstof*. Unpublished doctoral dissertation, University of Utrecht, Utrecht, Netherlands.

Van Hiele, P.M. (1986). *Structure and Insight: A Theory of Mathematics Education.* New York: Academic Press, Inc.

Van Hiele-Geldof, D. (1957). *De didaktick van de Meetkunde in deerste klass van het.* Unpublished doctoral dissertation, University of Utrecht, Utrecht, Netherlands.

Van Hiele-Geldof, D. (1984). *The didactic of geometry in the lowest class of secondary school.* Washington D.C.: National Science Foundation.

Webb, N. (2004). *Depth-Of-Knowledge Levels for Four Content Areas.* Retrieved February 1, 2010. http://facstaff.wcer.wisc.edu/normw/All...2032802.doc.

Williams, S. (2001). *Understanding Depth of Knowledge.* Retrieved July 20, 2011. www.maea.net/FADOK.ppt.

# Appendix A

## Surface Area and Volume Assessment

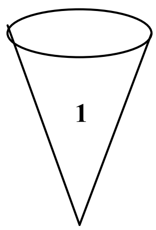
## (SAAVA)

Name: ________________________

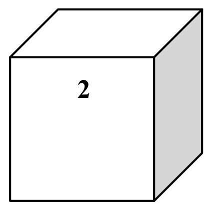
1. Name the shapes below and match the formula required to compute their respective volume and surface areas. Formulas may be used more than once.

a) $\pi r^2$ b) $bh$ c) $Bh$ d) $\frac{1}{3}Bh$ e) $S = 2B + Ph$

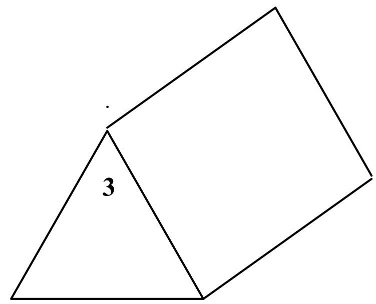
f) $2\pi r$ g) $\frac{4}{3}\pi r^3$ h) $S = B + \frac{1}{2}Pl$ i) $S = 4\pi r^2$



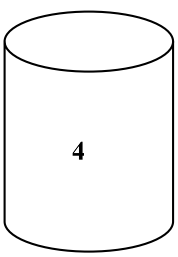

Name:
Volume:
Surface Area:



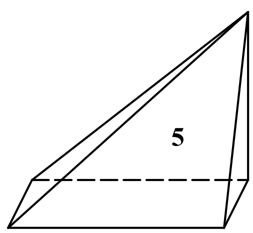

Name:
Volume:
Surface Area:



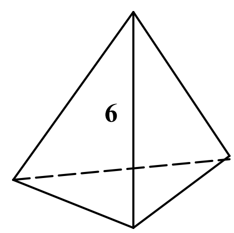

Name:
Volume:
Surface Area:



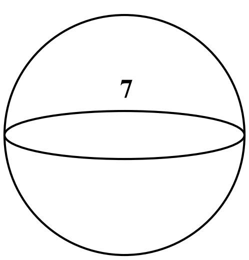

Name:
Volume:
Surface Area:



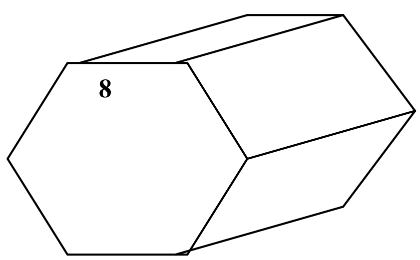

Name:
Volume:
Surface Area:




Name:
Volume:
Surface Area:



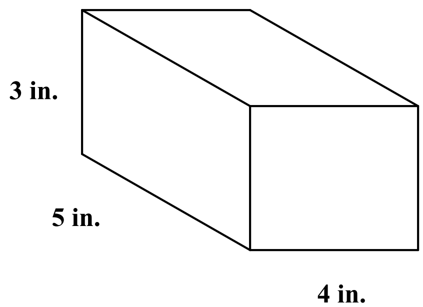

Name:
Volume:
Surface Area:



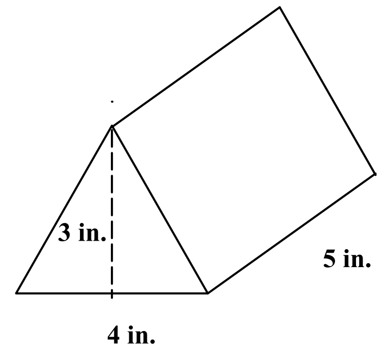

Name:
Volume:
Surface Area:

2. Calculate the volume and surface area of each solid.

a)



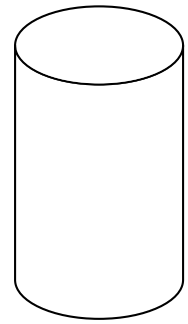

b)




c) Compare the volumes of the two solids above. What is the same and what is different? Use pictures, words, ratios, etc. in your explanation and be specific.

3.

a) Explain how to determine the surface area of the figure using words, pictures and/or equations.



b) Determine the height of the figure if its volume measures 21,205.8 $m^3$ and the diameter of the base is 30 m.

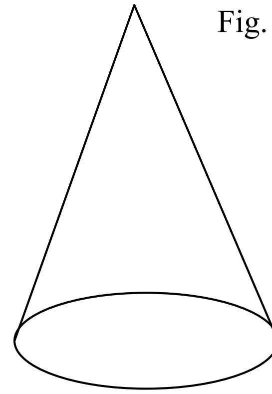
---

4. Explain how doubling the radius of a cylinder changes its volume. Use pictures, words, or formulas in your explanation and be specific.

5.

a) Explain how to determine the volume of figure *a* using words, pictures, and/or equations.

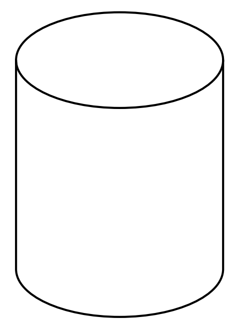
Fig. *a*



b) Label the figure *a* with the measurements needed to determine the surface area and volume of the figure.

c) What would it take for the volume of figure *b* to be equal to figure *a*? (*use words, pictures, and equations in your explanation)*

Fig. *b*



d) Would the surface areas of the two figures be equal if their volumes were equal? (use words, pictures, and equations in your explanation)

Appendix B

SAAVA Scoring Guidlines