The Beginnings of graph theory

Leonhard Euler, Seven Bridges, and A New Field of Study

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Figure 1: Map of the City of Königsberg

Introduction

There are many names that dominate the world of mathematics, from Euclid to Fermat to Riemann. One name though pervades nearly every field of study in mathematics, that of Leonhard Euler. Euler is known not only for his innovation and creativity, but also for the sheer volume of high quality work. Within this collection of work we can actually see the informal introduction of a new field of mathematics, the pre-creation if you will of graph theory. Euler, in his paper which appeared in *Commentarii Academiae Scientiarum Imperialis Petropolitane*, published in 1736, tackles the Bridges of Königsberg Problem, which can be considered as "the starting point of modern graph theory." [1] The problem is stated thusly:

> In the town of Königsberg there were in the 18th century seven bridges which crossed the river Pregel. They connected two islands in the river with each other and with opposite banks. The townsfolk had long amused each other with this problem: Is it possible to cross the seven bridges in a continuous walk without recrossing any of them.[2, p. 52]

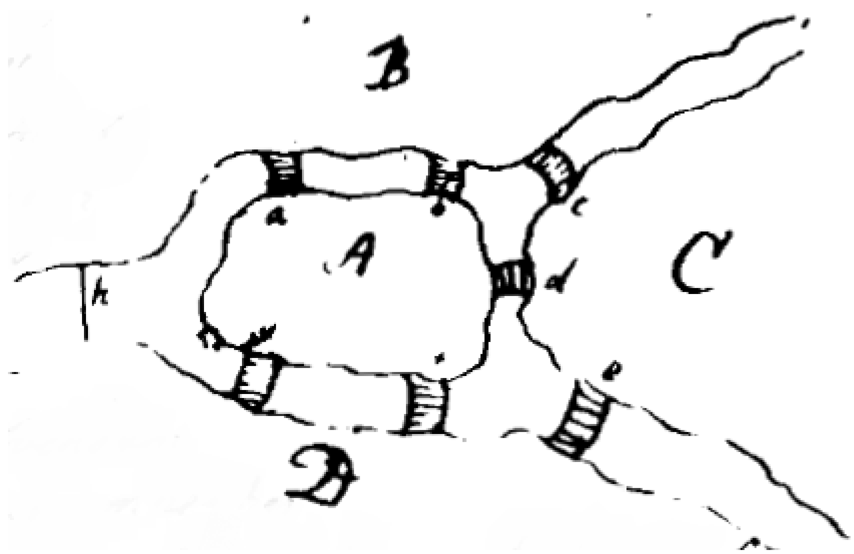

Figure 2: Visual Representation of the problem, drawn by Leonhard Euler in his paper *Solutio Problematis ad Geometriam Situs Pertinentis* (The Solution of a Problem Relating to the Geometry of Position).

The Bridges of Königsberg Problem

This problem at first seems to be a simple logic puzzle, a belief that Euler himself had, as can be seen in his communications with his friend, the Mayor of Danzig Karl Leonhard Gottleib Ehler [4] as follows:

> Thus you see, most noble Sir, how this type of solution bears little relationship to mathematics, and I do not understand why you expect a mathematician to produce it, rather than anyone else, for the solution is based on reason alone, and its discovery does not depend on any mathematical principle. Because of this, I do not know why even questions which bear so little relationship to mathematics are solvcd[sic] more quickly by mathematicians than by others.[4]

This did not stop Euler from not only presenting a definite solution to the problem but also designing a proof of his results. This paper ended up being one of the corner stones of graph theory. In this proof Euler considered a general example of the problem, suppose we have x islands, or vertices, and y bridges or edges connecting these landmasses. We can see that in order to “visit” an island(vertex) there must be an even number of bridges(edges), connecting that island(vertex) to the others, and this must be true for at most two of the islands, where the starting and ending points are positioned[5]. We now refer to graphs of this form as traversable or Eulerian graphs, the path taken as a traversable path.

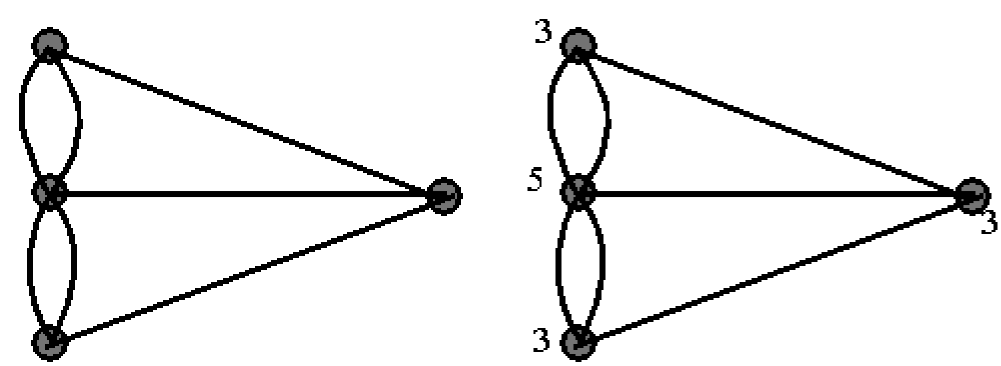

Figure 3: Graphical (multigraph) Representation of The bridges of Königsberg

We can see from the figure above that the solution can easily be seen if we are to utilize a vertex-edge graph, a model of Euler's design which we still use today, we can clearly see that in the original Königsberg Bridge map [Figure 2] there are more than two vertices that have an odd degree, in fact every vertex is of odd degree. Thus there cannot exist a path that can be taken to cross each bridge crossing the river Pregel exactly once.

The solution that Euler found, with the introduction of vertex-edge graphs helped to further a field that was not to be formally discovered for another two centuries, and is now one of the more powerful tools we have in the study of discrete mathematics. The study of graph theory also lends itself heavily to a multitude of fields, from applied mathematics to levels as high as theoretical physics where we can see the use of graph theory in string theory.[3]

Research and Discovery

In researching the Euler's method of finding the solution of the Königsberg Bridge problem we can see that graph theory was not a formally recognised field of mathematics until the 1930s when Dr. Konig began to analyse graphs in a systematic manner [6, (as cited in Gardner 1984, p.91)], even though Euler's paper, *Solutio Problematis ad Geometriam Situs Pertinentis* is considered at least a corner stone of the field if not the first publication in the field[1], nearly two hundred years before formalization of the field.

The idea that at least at first Euler himself did not believe that the Königsberg Bridge problem was not a problem for which he, as a great mind in the world of mathematics, was required as the problem was essentially a logic puzzle which has a solution in which the "discovery does not depend on any mathematical principle...for the solution is based on reason alone."[4] Which is especially intriguing as Euler published the solution with proof shortly after the letter in which he spoke of the disdain he felt surrounding the problem. The creation of Eulerian Graphs and vertex-edge graphs also shows up as a component of string theory, the current "in" topic in Theoretical Physics.[3]

Euler, graph theory, and Secondary Educators in the Classroom

In analysing thehistory behind the Königsberg Bridge problem we can see that in the classroom the question "When will I ever use/need this?" Is not in any circumstance an unfounded question, even one of the most prolific mathematicians of all time has asked this question, about a solution that has not only intrigued students of nearly all ages for centuries, but has also helped to lead to the entire field of graph theory.

We can also see that even something that seems to be of little importance, in this case a logic puzzle, can lead to ground breaking ideas such as graph theory and string theory. Using this knowledge we can let our students know that even if something seems trivial and unimportant those little things, such as factoring, trigonometric identities, etcetera can lead to more important topics in mathematics, or even in real world applications.

Visual Representations

List of Figures

References

[1] Barnett, J. H. Early writings on graph theory:euler circuits and the kőnigsberg bridge problem. Colorado State University.

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[6] Weisstein, E. W. Graph. From MathWorld-A Wolfram Web Resourse.