Introduction/Overview:

When creating this learning progression, I find it most helpful to choose a chapter from my Holt textbook and discuss the CCSS standards that will be addressed and shed light on what will need to be added to better align to the CCSSM. I've taken a deeper look at chapter 4, Triangle Congruence, from the Holt textbook. The text is lacking a connection between Transformations and Congruence. My plan for this chapter was to shed light and incorporate as many uses of transformations in relation to congruence, since the CCSS is highly focused on learning congruence in terms of transformations.

Learning Progression:

Chapter 4 - Triangle Congruence
4.1   Classifying Triangles
4.1A   Congruence and Transformations
4.2   Angle Relationships in Triangles
4.3   Congruent Triangles
4.4   Triangle Congruence: SSS and SAS
4.5   Triangle Congruence: ASA, AAS, and HL
4.6   Triangle Congruence: CPCTC
4.7   Isosceles and Equilateral Triangles

CCSSM Critical Area #1 (relating to congruence):
In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof.

> ❖ **Introduction:** To start off the unit, we will review the basic sections provided by the Holt textbook: classifying triangles, angle relationships in triangles. This will set the stage for the more rigorous ideas of congruence and proofs. The introduction should also include a brief review and formal definitions of the three rigid motions: transformations, reflections and rotations (G.CO.4). Dilations should also be mentioned, but it is important to clarify a dilation is not an isometry (when $k \neq 1$).

Representing Transformations in the Coordinate Plane

TRANSFORMATION	COORDINATE MAPPING AND DESCRIPTION
Translation	$(x, y) \rightarrow (x + a, y + b)$ Translation a units horizontally and b units vertically
Reflection	$(x, y) \rightarrow (-x, y)$ Reflection across y-axis $(x, y) \rightarrow (x, -y)$ Reflection across x-axis
Rotation	$(x, y) \rightarrow (y, -x)$ Rotation about (0, 0), 90° clockwise $(x, y) \rightarrow (-y, x)$ Rotation about (0, 0), 90° counterclockwise $(x, y) \rightarrow (-x, -y)$ Rotation about (0, 0), 180°
Dilation	$(x, y) \rightarrow (kx, ky), k > 0$ Dilation with scale factor k and center (0, 0)

❖ **Congruence:** Since students are already familiar with the term "congruent" (they have learned what congruent segments and angles are), I would discuss the criteria for triangles to be congruent. As we first start discussing triangle congruence, we set the stage by needing all three angles and all three sides of a triangle to be congruent in order for the whole triangle to be congruent. Transformations should be used to give meaning to the idea of congruency, discussed seamlessly throughout the unit. The Holt McDougal textbook is lacking in the integration of transformations and congruency. Be sure to discuss the relationship and stress the idea of rigid motions preserving angles and lengths (G.CO.6).
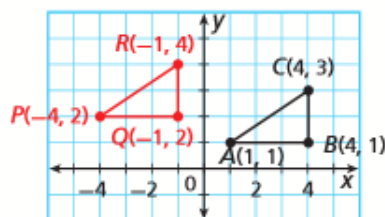
Determining Whether Figures are Congruent

Determine whether the polygons with the given vertices are congruent.

A $A(1, 1)$, $B(4, 1)$, $C(4, 3)$
    $P(-4, 2)$, $Q(-1, 2)$, $R(-1, 4)$

The triangles are congruent because $\triangle ABC$ can be mapped to $\triangle PQR$ by a translation:
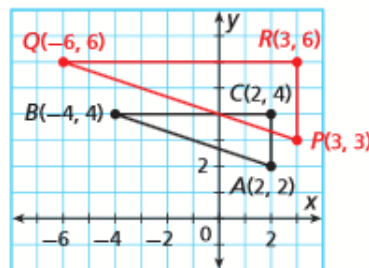
$(x, y) \rightarrow (x - 5, y + 1)$.

**B** $A(2, 2)$, $B(-4, 4)$, $C(2, 4)$
    $P(3, 3)$, $Q(-6, 6)$, $R(3, 6)$

The triangles are not congruent because $\triangle ABC$ can be mapped to $\triangle PQR$ by a dilation with scale factor $k \neq 1$:

$(x, y) \rightarrow (1.5x, 1.5y)$.

Once the idea of congruent triangles is established, you can begin to talk about the Congruent Postulate(SSS) and congruence Theorems (SAS, ASA, AAS, and HL). In fact, these Postulates and Theorems should be introduced using transformations (G.CO.8). It is important to discuss that using these Theorems, it guarantees a unique triangle. There is a great interactive website that will demonstrate these Theorems: http://illuminations.nctm.org/ActivityDetail.aspx?id=4. For example, given two sides and the angle between them, you try to construct a triangle that is not congruent, and it is not possible! Using this will help students see the significance of our Theorems and how they are all connected.

Using the diagram to the right, you can discuss how the SSS Postulate can be use to show the two triangles are congruent. You can also reason in terms of transformations, using a rotation and translation or a combination of reflections.
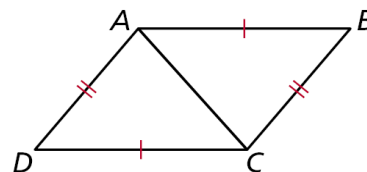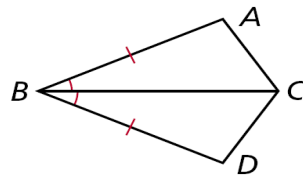
Again, you can discuss how a simple reflection guarantees that the two triangles are congruent, or you can use the SAS Postulate.

When introducing the Congruence Postulates/Theorems, it is important to talk about why AAA and SSA do not work. You could challenge students to provide a counter example of why AAA will not work (have them draw two triangles with congruent angles, that are not congruent - the side lengths will be proportional, but not congruent). It is slightly less obvious why SSA will not work (it does not guarantee that a unique triangle will be drawn). Instead of challenging students to discover this on their own, it can be a class activity. One way of explaining this to students is with the following graphic and the idea that SSA affords the possibility of creating zero, one or two triangles.
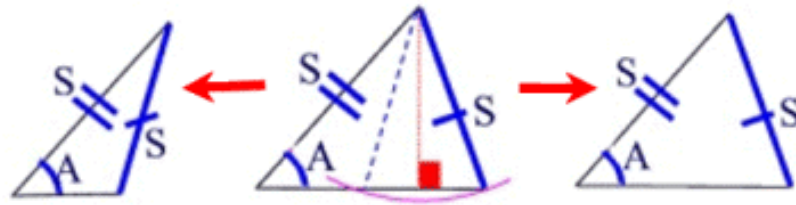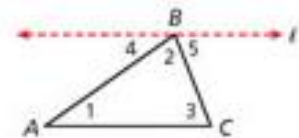
There is a great interactive website to help students visualize why SSA is not a Congruent Postulate (same website mentioned above) :
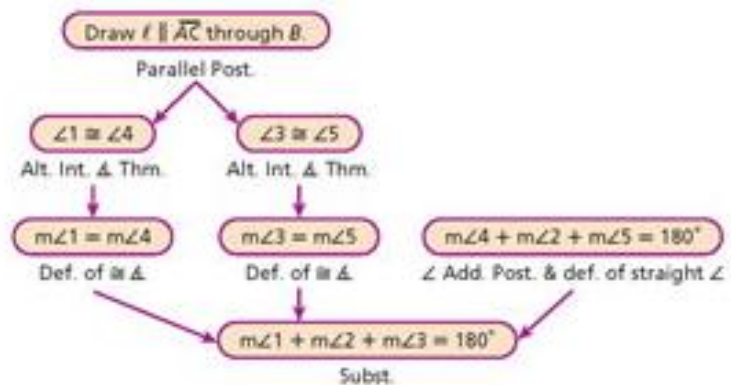http://illuminations.nctm.org/ActivityDetail.aspx?id=4
When discussing why SSA will not work, you can talk about how HL is just a special case of SSA. This would be a great time to remind students that when you have the right angle, it guarantees a certain relationship between the sides of a triangle.

❖ **Proofs**: There are several proofs that come into play in this critical area. Before congruence is established, there are a couple theorems in the Holt text that are very important. Firstly, is the Triangle Sum Theorem, stating that the interior angles of a triangle add up to 180 degrees. (G.CO.10) Below is a flowchart proof proving the Triangle Sum Theorem.

Given: $\triangle ABC$
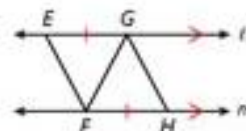Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Proof:

Draw $\ell \parallel \overline{AC}$ through B.
Parallel Post.

$\angle 1 \cong \angle 4$
Alt. Int. ∠ Thm.

$\angle 3 \cong \angle 5$
Alt. Int. ∠ Thm.

$m\angle 1 = m\angle 4$
Def. of ≅ ∠

$m\angle 3 = m\angle 5$
Def. of ≅ ∠

$m\angle 4 + m\angle 2 + m\angle 5 = 180°$
∠ Add. Post. & def. of straight ∠

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$
Subst.

Once congruence is
established, there are proofs
using SSS, SAS, ASA, AAS,
and HL to prove triangles are
congruent.

Proving Triangles Congruent

Given: $\ell \parallel m$, $\overline{EG} \cong \overline{HF}$
Prove: $\triangle EGF \cong \triangle HFG$
Proof:

| Statements	Reasons
1. $\overline{EG} \cong \overline{HF}$	1. Given
2. $\ell \parallel m$	2. Given
3. $\angle EGF \cong \angle HFG$	3. Alt. Int. ∆ Thm.
4. $\overline{FG} \cong \overline{GF}$	4. Reflex Prop. of $\cong$
5. $\triangle EGF \cong \triangle HFG$	5. SAS *Steps 1, 3, 4*

Conclusion:
Though all teachers are different have different philosophies, I think a variety of direct
instruction, exploration, scaffolding, group work and independent work should be used in this
unit. Classroom assessments would include in class assignments, homework, quizzes, and a test.
The following assessments would be great benchmark assessments for the standards addressed in
this unit. All of these assessments are taken from Illustrative Mathematics (links included).

❖ **G-CO Are the Triangles Congruent?**
   http://www.illustrativemathematics.org/illustrations/33

   This activity will assess students understanding of congruence
   including reasoning using Congruence Postulates. Students are
   asked to determine whether or not the triangles in the diagrams
   are congruent. You could extend this activity to include a proof
   of congruence (if their answer is yes) or an explanation of why the triangles are not
   congruent. One of the diagrams is shown to the right. Students are told that ABCD is a
   parallelogram.
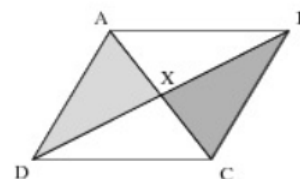
❖ **G.CO Reflections and Isosceles Triangles**
   http://www.illustrativemathematics.org/illustrations/983

   This activity will assess students understanding of reflections,
   congruence, and special triangles. Students are asked to show
   that points of an isosceles triangles are mapped onto themselves
   after a reflection in the y-axis. The diagram is shown to the
   right. Students are asked to show several things, including that
   reflection in the line L will map triangle ABD onto triangle
   ACD.
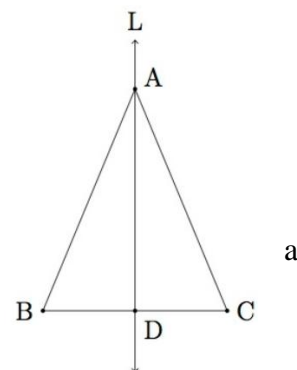
❖ **G.CO Reflections and Equilateral Triangles**
   http://www.illustrativemathematics.org/illustrations/982

   This activity assess similar ideas as the one above, but in
   relation to equilateral triangles. Students are asked to
   describe the reflections that map triangle ABC onto itself
   and then prove some properties of those lines.