**Progression for High School Common Core State Standard N-VM.6-12, using *Advanced Mathematics* by Richard Brown (Houghton Mifflin), Mrs. Lefebvre’s classroom**

**Overview**

Matrices are a topic that is not introduced until the high school level. Although some instructors may teach matrices incidentally as one of several strategies that can be used to solve systems of linear equations in multiple variables, many students have not seen matrices prior to their introduction in this unit.

This learning narrative is designed to cover the high school level Common Core State Standard (CCSS) N-VM.6 through 12. This section contains all of the standards relating to matrices that are included in the CCSS. The overarching theme for this section is that students are to perform operations on matrices and use matrices in applications.

It should be noted that each of these standards is considered important for students who plan to take advanced courses in mathematics and/or go on to college. In addition, matrices are a common area where modeling may be introduced and used, a practice which is emphasized and encouraged by the standards and mathematical education experts.

While matrix mathematics is not difficult in terms of the actual computations, there are several areas within the topic that often cause initial student confusion. Most students are familiar with the terms “row” and “column” and have no difficulty with identifying rows and columns within matrices; however, students often have trouble remembering which comes first when describing the dimensions of a matrix. Students also often find the traditional method of identifying a specific number

In the context of the CCSS, modeling means using the mathematical methods and techniques learned to solve real-world type problems, using actual data when possible.



within a matrix as being in the ith row and jth column incomprehensible in the beginning. If teaching this method of identification, it is probably beneficial to provide multiple examples and give students practice before proceeding on to other topics.

Another area that often results in student error occurs in matrix multiplication. Students who have been inculcated in algebraic methods have ingrained in their heads the concept that multiplying A times B is the same as multiplying B times A; in other words, the order in which the terms are listed makes no difference. This is almost never the case with matrix multiplication, which often causes students some frustration. It is important, therefore, to emphasize this concept on multiple occasions.

Finally, it is important to note that there are differences between matrices and sets. While both could loosely be termed collections of objects, which are usually number values, the order of these objects does not matter in sets, while the arrangement of the values in matrices has specific meaning. One way to emphasize this to students is to point out the difference in notation. Also, it can be shown that while the values in a matrix can be components of a set and therefore can be put into set notation, the reverse is not possible without additional information; that is, we cannot take set values and put them into a matrix in the proper order unless we know something about the arrangement of the values.



[A]\*[B] ≠ [B]\*[A]!

N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

Curly braces, { }, indicate sets,

brackets, [ ], indicate matrices

**Performing Operations Using Matrices**

As previously mentioned, the actual calculations required for matrix mathematics are simple, often involving only the addition, subtraction, or multiplication of whole numbers. The tricky part, as far as students are concerned, is knowing the proper order (i.e. which numbers) to use to perform the calculations, and where to place the answers within the resulting matrix once they have calculated them. This is especially true when multiplying two matrices together. When introducing the various calculations that can be performed using matrices for the first time, it might be easier on students to work initially with scalar multiplication, rather than beginning with matrix addition. Students should already be quite familiar with using the distributive property, so it will not be a stretch to apply the concept to a new setting. Students will thus be able to focus on getting comfortable with unfamiliar format, without the added complication of learning new techniques. And, if the course has been taught following the CCSS in the order they are presented in the document, students will already have learned to multiply vectors by scalars, so moving to multiplying multiple-column matrices by scalars will be a very small step indeed.

As with scalar multiplication, students may already be familiar with the rudiments of adding and subtracting matrices if they have already had instruction in performing these operations on vectors. The key concepts for this area are two-fold: understanding that dimensions of the matrices involved determine whether or not the operation is viable, and knowing proper placement of the calculated values within the resultant matrix.

Students generally have no trouble with the concept that matrices must be of the same dimensions in order to add or subtract them from one another. Trouble begins when we

N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

Vectors are usually written in matrix form, using one column and with the number of rows equal to the dimensional space in which the vector exists.

N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

change the rules on them when it comes to multiplication. It is not unusual for students to be able to glibly recite the rule, “the number of columns in the first matrix must equal the number of rows in the second matrix,” yet still try to multiply matrices of inappropriate dimensions. It is therefore desirable to show multiple examples when explaining this topic, and perhaps even provide practice that consists only of correctly identifying whether or not matrices with demonstrated dimensions may be multiplied together, without requiring that the actual calculations be performed.

One final area that the CCSS cover in regards to matrices includes the concepts of zero matrices and identity matrices and their similarity to 0 and 1 in the real number system, as outlined in standard N-VM.8. Often this makes intuitive sense to students, although the use of the identity matrix in multiplication may need to be shown more than once in order for students to make the visual connection.

The second half of this standard tends to be more confusing for students. It may well be easier for the teacher to demonstrate, and the students to comprehend, that nonzero determinants lead to multiplicative inverses for matrices rather than showing that multiplicative inverses mean determinants are nonzero, especially since the determinant itself is a new concept to be presented and learned.

**Using Matrices in Applications**

The CCSS matrix cluster includes three standards that require application of matrix mathematics. N-VM.11 involves working with vectors, including multiplication of a vector by another vector or matrix to produce a new vector (transformation). A common real-world, albeit a bit simplified, exercise that relates to this standard is the multiplication of a vector representing a projectile by another vector representing the force of gravity, to determine the resultant

N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.



N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

direction and present the answer in vector form.

At this point in their mathematical education, most students have studied the transformations within the plane of two-dimensional shapes. Matrices can now be introduced as succinct ways to sum up the actions taken to produce transformations. Indeed, the use of 2x2 matrices to indicate the operations should help reinforce the fact that transformations within the plane are taking place in two-dimensional space.

However, the greatest opportunity for demonstrating the applicability of matrix mathematics may be when using them to model real-life situations. There are many areas that lend themselves to this kind of application; in particular, problems encountered in engineering and business are often best solved by using matrix mathematics. Without too much of a stretch, it should be possible for teachers to find situations that students can connect with. Students who attend schools in an urban setting might use matrices to solve traffic flow problems, while those who live in rural, agricultural settings might solve irrigation flow problems. The concept of circuitry in electronics is a classic topic in matrix mathematics and would be applicable to students interested in multiple vocational areas. In fact, the possibilities are nearly limitless when it comes to finding and using matrices in ways that model real-world applications. Let your students’ interests be your guide to finding situations and applications for matrix mathematics, and you are almost guaranteed to have an engaged classroom where students will willingly participate and truly learn the concepts being presented by engaging with the topic in a way that is meaningful for them.

N-VM.12. (+) Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

N-VM.6. (+) Use matrices to represent and manipulate data, e.g., as when all of the payoffs in a game are doubled.