High School: Functions
(Interpreting Functions)

Introduction:
This unit focuses the cluster of *Understanding the concept of a function and use function notation* in the Interpreting Function domain of the Common Core State Standards for Mathematics. In this unit students will be introduced to functions and their properties. By the end of the unit students will be able to identify the domain and range of a given functions, in addition they will understand the one-to-on relationship between numbers in the domain and numbers in the range [HSF-IF.A.1]. When speaking of functions students will be able to use the correct vocabulary, likewise, they will use correct notation when referencing them on paper. Students will also be able to evaluate a given function with given an input value for the function[HSF-IF.A2]. This unit will also introduce student to common functions, knows as parents functions, that they will often see throughout the rest of their mathematical careers. The last things students will learn in this unit is how to recognize that sequences are functions and how to express some sequences in function notation[HSF-IF.A3].

HSF-IF.A.1 – Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one elements of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

HSF-IF.A.2 – Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.

HSF-IF.A3 – Recognize that sequences are functions, sometimes defines recursively, whose domain is a subset of integers. *For example, the Fibonacci sequence is defined recursively by f(o) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n≥ 1.*

The book the learning progression will use is *Holt McDougal Algebra 2*. The way the book is set up, the learning progression will utilize two different chapter of the book. The first is Chapter 1: Foundations for Functions. Three lessons from this chapter will be used to help meet the learning standards; 1-6 Relations and Functions, 1-7 Function Notation and 1-9 Introduction to Parent Functions. The second chapter that will be used is Chapter 12: Sequences and Series. For this chapter only the first section (12-1 Introduction to Sequences) will be used. Although it may seem a little inconvenient to jump around in the textbook in the way mentioned about, it is the best approach to cover this cluster of the CCSS in a logical sequence.

What is a Function?
For this activity students will be taking interactive notes to understand that a function is a system in which for every input there is exactly one output. These notes will also help the students to understand that the set of inputs is called the domain and the set of outputs is called the range [HSF-IF.A.1].

HSF-IF.A.1 – Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one elements of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

Before introducing students to a function, they first need to be introduced to the idea of a relation. In their notes students will write the definition of relation given on p. 44 of their textbooks. Below this students will glue a foldable into their notebooks that looks like the below diagram.

N Numerically	A Algebraically
G Graphically	V Verbally (Sentence)

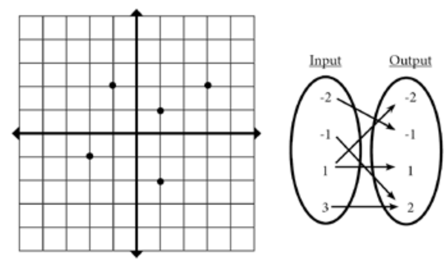
Each of these four sections will be a flap that will fold outward. Under each flap students will write/draw examples of relation represented in the format indicated on the front of each flap. For example: under the N flap there would be examples of a set of ordered pairs and then and Input-Output table for this set, under the G flap there would be a coordinate plane with a few points plotted and then a mapping diagram of these points. Some of these examples should be functions and some should not be functions. There is no need to point this out to students at this time as it will be discussed at a later time. Below this foldable or on the next page students will now right the definitions for domain and range found on p. 44 of their textbooks. To help students remember the domain and range the teacher will introduce students to the acronym DIXROY which stands for Domain Inputs X-Coordinates Range Outputs Y-Coordinates. A simple table can be drawn in the students notes that that looks like this:

D Domain	I Input	X X-Coord.	R Range	O Output	Y Y-Coord.
The **D**omain of a relation is the set of all **I**nputs or **X**-Coordinates.			The **R**ange of a relation is the set of all **O**utputs or **Y**-Coordinates.		

Following this chart, students will complete Example 1 from their books (p. 44) in their notes. At this point teacher will need to decide if the students need another example or if they are ready to move on.

Now it is time to introduce students to functions. On the top of the next clean page in their notebooks students will write down the definition of a function found on p. 45 of their textbooks. At this point the teacher should direct student back to the NAGV foldable done earlier in class.

Example under the G flap:



Example 1 from textbook:

Identifying Domain and Range

Give the domain and range for the relation shown.

First-Class Stamp Rates						
Year	1900	1920	1940	1960	1980	2000
Rate (¢)	2	2	3	4	15	33

List the set of ordered pairs:

{(1900, 2), (1920, 2), (1940, 3), (1960, 4), (1980, 15), (2000, 33)}

Domain: {1900, 1920, 1940, 1960, 1980, 2000} *The set of x-coordinates*

Range: {2, 3, 4, 15, 33} *The set of y-coordinates*

Looking at these relations discuss which examples are function and which are not and why this is so. After this students should be back on their page of notes that has the definition of a function on the top. At this time the students should divide the rest of the page into two columns; the left hand column titled Function and the right Not A Function. At this time the teacher will pass out a page of relations represented in different ways (coordinate planes, mapping diagrams, sets or ordered pairs, tables). About 6 or so should be function and about the same number should not be functions. Students will then work with a partner or small group to cut out and glue these relations into the appropriate columns in their notes (function or not a function). When this is happening the teacher should be walking around the classroom to make sure students are not incorrectly gluing things into their notebooks and to check for understanding. After most of the class as finished, the teacher should go over an example with the class. This will allow those who are finishing up time to complete theirs and time for the others to check their table. Following this activity the students will complete Example 2 and 3 from their textbook (p. 45) in their notes. At this point teacher will need to decide if the students need another example or if they are ready to move on.

To allow students to practice this and show their understanding of functions and their domain and range [HSF-IF.A.1] homework from the book will be assigned. Students should complete problems 1-19 in their textbook (p.47-48). These questions will be similar to the three examples done in class. They will also be similar to the questions presented on the end of the unit assessment to test students understanding of functions.

Note that this is a rather long activity that requires a lot of cutting and glue in notebooks. Depending on mathematical comprehension of students, size of class, and length of class period this activity might take two days.

Function Notation

Before this activity on using function notation the teacher will have needed to gone over notes with the students. These notes include a definition on function notation found on p. 51 of the textbook and Example 1 and 3. Example 1 (p. 51) focuses on evaluating functions in function notation [HSF-IF.A.2] and by looking at a graph [HSF-IF.A.1]. Example 3 (p. 53) is a real world example of a situation that can be modeled by

Example 2 from textbook:

Determining Whether a Relation Is a Function

Determine whether each relation is a function.

A

Instant Rice Cooking Times				
Servings	2	4	6	8
Cooking Time (min)	5	8	10	11

There is only one cooking time for each number of servings. The relation from number of servings to cooking time is a function.

B from last name to Social Security number

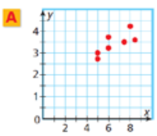
A last name, such as Smith, from the domain would be associated with many different Social Security numbers. The relation from last name to Social Security number is not a function.

Example 3 from textbook:

Using the Vertical-Line Test

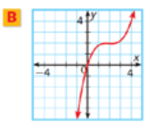
Use the vertical-line test to determine whether each relation is a function. If not, identify two points a vertical line would pass through.

A



This is *not* a function. A vertical line at $x = 6$ would pass through $(6, 3.25)$ and $(6, 3.75)$.

B



This *is* a function. Any vertical line would pass through only one point on the graph.

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HSF-IF.A.2 – Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.

Example 1 from textbook:

Evaluating Functions

For each function, evaluate $f(0)$, $f\left(\frac{1}{2}\right)$, and $f(-2)$.

A $f(x) = 7 - 2x$

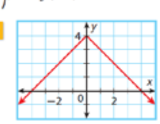
Substitute each value for x and evaluate.

$f(0) = 7 - 2(0) = 7$

$f\left(\frac{1}{2}\right) = 7 - 2\left(\frac{1}{2}\right) = 6$

$f(-2) = 7 - 2(-2) = 11$

B



Use the graph to find the corresponding y-value for each x-value.

$f(0) = 4$ $f\left(\frac{1}{2}\right) = 3\frac{1}{2}$ $f(-2) = 2$

a function [HSF-IF.A.2].

For this activity the teacher will need to have predetermined two or three function to use. For each function the teacher must pick a set of input and output value that totals the number of students in the class. For example, if one of the functions were $f(x) = x^2 - 2$ and there were six student in the class the following values might be chosen:

x	$f(x)$
-3	7
-1	-1
0	-2
2	2
4	14
5	23

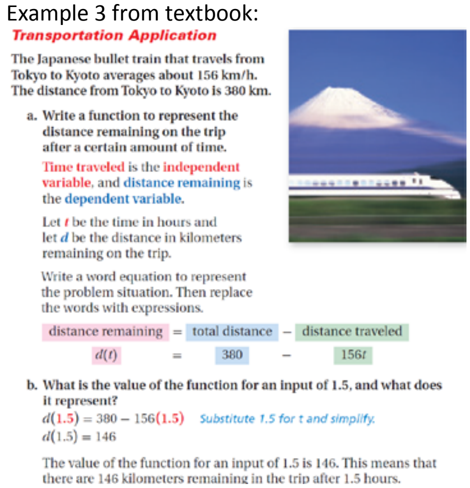
Note: if there are two different input values that yield the same output value they should not both be chosen for this activity to work.

Each function should also be assigned its own color. Using twice the number of students in the class the teacher should cut small business card sized pieces of color paper. One set of cards for each function. In the example of six students there should be twelve cards in each color (a different color or each function). On half of these cards the teacher will need to write $x = (input\ value)$ choosing input values from the predetermined list. On the other half of the cards the teacher need to write $f(x) = (output\ value)$ choosing output values from the predetermined list. For the example of $f(x) = x^2 - 2$, the color assigned might be blue and the following cards made.

$x = -3$ $f(x) = 7$

During the activity the teacher should write all the function on the whiteboard in there designated color. And each student should get a card with an input value and a card with an output value in each color. To start the activity the teacher must chose a function to start with and a student to start. The student who starts will read their input card for the chosen function. The other students will need to do the math and figure out who has the output card that matches. Once the student with the output card is identified they will then read input card and the process with continue. This will go on until it gets back to the student who started. Once back at the beginning (every student has gone) the

Example 3 from textbook:



Transportation Application

The Japanese bullet train that travels from Tokyo to Kyoto averages about 156 km/h. The distance from Tokyo to Kyoto is 380 km.

a. Write a function to represent the distance remaining on the trip after a certain amount of time.

Time traveled is the independent variable, and distance remaining is the dependent variable.

Let t be the time in hours and let d be the distance in kilometers remaining on the trip.

Write a word equation to represent the problem situation. Then replace the words with expressions.

distance remaining	=	total distance	−	distance traveled
$d(t)$	=	380	−	$156t$

b. What is the value of the function for an input of 1.5, and what does it represent?

$d(1.5) = 380 - 156(1.5)$ *Substitute 1.5 for t and simplify.*

$d(1.5) = 146$

The value of the function for an input of 1.5 is 146. This means that there are 146 kilometers remaining in the trip after 1.5 hours.

whole activity starts over with a new function.
This activity will help students to get practice using function notation to evaluate functions[HSF-IF.A.2]. This activity can also be easily adapted to help students use function notation to model real world problems[HSF-IF.A.2]. Instead of cards with input and output values there could be cards with a real world situation and cards with a function than models it. For example:

GoLean vitamins are sold by mail order only. They cost $19.99 per bottle plus $4 shipping and handling.	$f(x) = 19.99x + 4$

To allow for more practice homework from the textbook will be assigned; problems 1-4, 11-17 and 21-22 (p. 54-55). These questions will be similar to the two examples done in class. They will also be similar to the questions presented on the end of the unit assessment to test students understanding of function notion.

Sequences are Functions

The textbook used in the learning progression does not have a section that presents sequences as function[HSF-IF.A.3]. Because of this, the teacher must use the textbook to introduce student to sequences and then modify examples and problems in the textbook in order for students model sequences in function notation[HSF-IF.A.3].
The teacher should lead students in notes using the textbook as a guide. This way students will get the following vocabulary terms presented on p. 862: sequence, term of the sequence, infinites sequence, finite sequence and recursive formula. After the vocabulary is out of the way, the notes should go straight into examples. Example 1 on p. 862 asks the student to find the first 5 terms of the sequence. This can be modified by the teacher to first have the students model the sequences in function notation, and then find the first 5 terms. The example can be first done by the teacher with the two extra examples (1a and 1b) to be done by the teacher with more student involvement. At this time the teacher will need to decide if the students need more examples or if they are ready to move on. If they are ready to move on, the next example the students should see is Example 3 on p.863. Like example one there

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HSF-IF.A3 – Recognize that sequences are functions, sometimes defines recursively, whose domain is a subset of integers. *For example, the Fibonacci sequence is defined recursively by f(o) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n≥ 1.*

Example 1 from textbook:

Finding Terms of a Sequence by Using a Recursive Formula

Find the first 5 terms of the sequence with $a_1 = 5$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$.

The first term is given, $a_1 = 5$.

Substitute a_1 into the rule to find a_2. Continue using each term to find the next term.

The first 5 terms are 5, 11, 23, 47, and 95.

n	$2a_{n-1} + 1$	a_n
1	Given	5
2	2(5) + 1	11
3	2(11) + 1	23
4	2(23) + 1	47
5	2(47) + 1	95

Example 1a and 1b from textbook:

Find the first 5 terms of each sequence.

1a. $a_1 = -5, a_n = a_{n-1} - 8$ **1b.** $a_1 = 2, a_n = -3a_{n-1}$

Example 3 from textbook:

Writing Rules for Sequences

Write a possible explicit rule for the nth term of each sequence.

A 3, 6, 12, 24, 48, ...

Examine the differences and ratios.

Ratios 2 2 2 2

Terms	3	6	12	24	48

1st differences 3 6 12 24

2nd differences 3 6 12

The ratio is constant. The sequence is exponential with a base of 2.

Look for a pattern with powers of 2.

$a_1 = 3 = 3(2)^0, a_2 = 6 = 3(2)^1, a_3 = 12 = 3(2)^2, \ldots$

A pattern is $3(2)^{n-1}$. One explicit rule is $a_n = 3(2)^{n-1}$.

B 2.5, 4, 5.5, 7, 8.5, ...

Examine the differences.

Terms	2.5	4	5.5	7	8.5

1st differences 1.5 1.5 1.5 1.5

The first differences are constant, so the sequence is linear.

The first term is 2.5, and each term is 1.5 more than the previous.

A pattern is $2.5 + 1.5(n - 1)$, or $1.5n + 1$. One explicit rule is $a_n = 1.5n + 1$.

Example 3a and 3b from textbook:

are two extra examples (3a and 3b) that allow for the students to practice. Also like example one the teacher will need to decide if the students need more examples or if they are ready to do this on their own.

Write a possible explicit rule for the *n*th term of each sequence.

3a. $7, 5, 3, 1, -1, \ldots$ **3b.** $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

To allow for practice recognizing sequences as functions [HSF-IF.A.3] homework from the textbook will be assigned; problems 2-4, 11-13, 16-18 and 22-24 (p. 865-866). Similarly to the example questions these questions will need to be modified to fit the need of the standard. These homework problems will also be similar to the questions presented on the end of the unit assessment to test student's ability to identify sequences as functions [HSF-IF.A.3].

HSF-IF.A3 – Recognize that sequences are functions, sometimes defines recursively, whose domain is a subset of integers. *For example, the Fibonacci sequence is defined recursively by f(o) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n≥ 1.*