Creating Equations in High School Algebra According to the Common Core Standards

For 9th and 10th grade using "*Algebra 1: Expressions, Equations, and Applications*" by Paul Foerster ©1999

V. Martinez

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Create Equations that Describe Numbers or Relationships

Preface

This is a collection of methods for teaching of the Algebra standards in the Common Core State Standards (CCSS) for high school. The target demographic for this learning progression is a school in Eastern Washington. The Algebra classroom will be 25-30 students varying from 9^{th} through 10^{th} grade. There are students who have special needs, thus lessons must allow for accommodations and modifications.

The textbook for this progression will be "Algebra 1: Expressions, Equations, and Applications" by Paul Foerster ©1999. The book chosen for this course does not go in the order that the standards are listed, but we will ensure to cover all 4 standards.

Students will need to know how to solve linear equations, and use basic algebraic axioms. The cluster would take approximately 2 weeks to cover. The common core standards for 8^{th} grade Algebra has left students off in a great place to pick up for high school Algebra. Amongst many others, they have learned the following useful standards for this cluster:

- CCSS.Math.Content.8.EE.C.7 Solve linear equations in one variable.
- CCSS.Math.Content.8.EE.C.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- CCSS.Math.Content.8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Every day will begin with an entry task. Students will begin the lesson by exploring the prior or day's learning objective.

Create equations and inequalities in one variable and use them to solve problems

Math.CCSS.Math.Content.HSA-CED.A.1

Include equations arising from linear and quadratic functions, and simple rational and exponential functions

By this point in a student's education, they have learned basics of inequalities, but need a reminder. Sometimes, a "DICTIONARY" may be helpful for a quick go-to when students need to look for a key-word or phrase. Something that looks like the one provided below may be useful.

Algebra Symbol	**Key Words**
= equals	1. all 2. equals 3. gives 4. is, are, was, were, will be 5. results 6. same 7. yields
< is less than	1. below 2. less than
≤ is less than or equal to	1. maximum of 2. not more than
> is greater than	1. greater than 2. more than 3. over
≥ is greater than or equal to	1. at least 2. minimum of 3. not less than
+ addition	1. add 2. and 3. combine 4. increase 5. more 6. plus

	7. raise 8. sum 9. together 10. total
– subtraction	1. decrease 2. difference 3. fewer 4. less 5. lose 6. minus 7. reduce
× multiplication	1. directly proportional 2. double(× 2), triple(× 3), etc. 3. group of 4. linear 5. multiplied 6. product 7. times
/ division	1. average 2. cut 3. divided by/into 4. each 5. inversely proportional 6. out of 7. per 8. pieces 9. quotient 10. ratio 11. share 12. split
x^n power	1. power 2. square (n = 2), cube (n = 3), etc.
n^x exponential	1. decays 2. doubles (n = 2), triples (n = 3), quadruples (n = 4), etc. 3. grows 4. rate of n per x

Once students get a hang of the basics, we can begin learning quadratic, rational and exponential functions to address all aspects of this standard.

We want to have students create an equation from a single variable. There are many ways to show this, students must be able to extract the information necessary to create an equation. Some examples of this would be Activities 1.A and 1.B.

The possible answers for Activity 1.A are:

(A) $A + 18 = 24$
(B) $A + A + A = 60$
(C) $60 - 18 = A - 24$
(D) $24 + 18 + A = 60$

The correct answer is D. The relation:
Hooper's class + Gomez's class + Anderson's class = 60 students
going on a field trip becomes an equation by changing the written descriptions into numbers and variables. Mrs. Hooper's class has 24 students and Mr. Gomez's class has 18 students giving
24 + 18 + Anderson's class = 60 students
going on a field trip. The number of students in Anderson's class is the unknown and must be represented by a variable like A for Anderson. That means:
$24 + 18 + A = 60$

For Activity 1.B, the possible answers would look something like:

A. $4x^2 + 3x^2 + 2x = 69$
B. $4x^2 - 3x^2 + 2x = 69$
C. $4x^2 + 3x^2 - 2x = 69$
D. $4x^2 - 3x^2 - 2x = 69$

The correct answer is *A*. Begin with the relation:
combine heart shape and small box = 69 chocolates total
The key words "and, "adding," and "add" all indicate summing the three relations for numbers of chocolates. This gives:
4x2 + 3x2 + 2x
Combining these gives the equation: *4x2 + 3x2 + 2x = 69.*

Activity 1. A

There are 60 students going on a field trip to the chocolate factory. The students are from three different classes. Mrs. Hooper's class has 24 students and Mr. Gomez's class has 18 students. Which of the equalities correctly describes the students and could be used to solve for how many students are from Mr. Anderson's class? (Let A = the number of students in Mr. Anderson's class.)

Activity 1. B

A heart shaped chocolate box is composed of one square and two half circles. The total number of chocolates in the box is calculated by adding the area of a square given by 4x2 and the area of a circle approximated by 3x2. The company plans to add a small additional box for a promotional campaign containing one row (2x) of chocolates. If the total combined heart shape and small box contain 69 chocolates, which of these equations could be utilized to solve for the number of chocolates in the small box (2x)?

Create equations in two or more variables to represent relationships between quantities

Math.CCSS.Math.Content.HSA-CED.A.2

Graph equations on coordinate axes with labels and scales.

This standard has two significant components. The first is translating word problems into equations with two or more variables. Translating word problems to create simple equations with two or more variables is not that different conceptually from creating equations with one variable. The biggest difference is that more complicated relationships may develop. This standard should be taught with the previous one.

The second component is creating graphs of equations on coordinate axes, which incorporates multiple skills such as visual perception, interpreting data, and synthesizing information. These types of graphs relate equations with multiple equations by relating one variable to another. For example. In the form y = mx + b, we can look at either x or y and any defined value for x will give us a defined value for y, and vice versa. Graphs can help visualize these relationships between variables and facilitate the connection of equations to the graphs that represent them.

Using the EXPLORATIONS section, show students what makes a tangent line. Have students come up to either a document cam or board to complete the tangent line or radii. Which will lead to Theorems 1 &2.

There are many possible examples of activities for this standard. Activity 2.A shows understanding of relationships and ratios.

The possible answers are:

(A) Ratio of nut chocolates to cordials = 1:3

(B) Ratio of nut chocolates to cordials = 2:3

(C) Ratio of nut chocolates to cordials = 1:2

(D) Ratio of nut chocolates to cordials = 2:1

The correct answer would be D, or "Ratios of nut chocolates to cordials -2:1" The relationship 2:1 is already expressed as a ratio;

ACTIVITY 2.A

The ratio of nut chocolates to cordials in an assortment box is 2:1. Which equality describes the contents of an assortment box?

so, this is just a simple equality of ratio of nut chocolates to cordials = 2:1.

Activity 2.B provides an example where students must consider inequalities. The possible answers would be:

(A) Chaperones required ≥ one for every 12 students

(B) Chaperones required ≥ three chaperones for 30 students and another chaperone for every 12 additional students

(C) Chaperones required ≤ one for every 12 students

(D) Chaperones required ≤ three chaperones for 30 students and another chaperone for every 12 additional students

The answer for this sort of problem lies in the fact that there are two different requirements for chaperones. Three are required for any field trip with up to 30 students. Then, for every 12 students over the 30-student mark, an additional chaperone is required. These two requirements need to be combined to calculate the minimum number of chaperones for a trip. This gives the answer B, where the inequality chaperones required ≥ three chaperones for 30 students and another chaperone for every 12 additional students.

ACTIVITY 2.B

The Fuzzlegump School for Gifted and Not-So-Gifted Children requires that three chaperones go on any field trip. More chaperones are required if there are more than 30 students on the trip. For every 12 additional students, another chaperone is required. Which of these inequalities describes the number of chaperones required?

Represent constraints by equations or inequalities

Math.CCSS.Math.Content.HSA-CED.A.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional cost constraints on combinations of different foods.

Students have already translated words into algebraic equations and have actually taken the time to solve the problem. Students now have to interpret the results. This standard is about three things:

1.Creating equations/inequalities or systems of equations/inequalities
2.Solving these equations/inequalities or system of equations
3.Interpreting the answer properly

To analyze problems in which multiple relationships affect multiple variables, students must be able to create systems of equations, solve them, and interpret the results appropriately.

In order to create systems of equations from word problems or other contexts, students need to be able to differentiate the relations and create equations for each. To support this, should already be able to create equations from word problems.

Creating equations from a word problem or similar context is a three-step translation process:

1.Translate the equality or inequality (=, <, >, ≤, or ≥)
2.Translate the operations (+, –, ×, ÷, x^n, n^x)
3.Translate the numbers and variables

Systems of equations are identified during step one of this process. Students need to be able to read a problem and identify how many equality and inequality relations are described. Then, they should write each down separately.

Once these are written down, students perform steps two and three (translating the operations, numbers, and variables) independently for each equation.

Hopefully, students already know how to solve systems of equations.

After all, it's necessary in order to interpret results from a set of equations.

Solving systems of equations can be done through substitution or adding the two equations together to cancel out one of the variables. The goal being to eliminate a variable so that we can solve for the other and then substitute that back in to find the value of the next variable.

Students tend to struggle to understand what an algebraic result means in the context of a word problem. An error that happens a lot is to report an answer based on a different variable in the problem. Writing down the variable information helps to prevent such errors. Interpreting results should be done through the algebra course. Eventually students will get to the point that they use whatever methods work best for them.

Explanation activity 3.A The height of h = 0 gives (-5t + 30) × (t + 2) = 0. This is possible if -5t + 30 = 0 or if t + 2 = 0. The equation -5t + 30 = 0 gives us t = 6. The equation t + 2 = 0 gives us t = -2. Both t = 6 and t = -2 are correct solutions to the equation. However, a negative flight time does not make sense. Only the t = 6 solution for a time of 6 seconds makes sense. The rocket hits the ground 6 seconds into the flight.

Activity 3.B could be used as a modeling problem where students can experiment with the marbles and see how many fit in the box and how to write this in an inequality. For instance, if Small boxes of chocolates contain 12 pieces and large boxes of chocolates contain 45 pieces; if someone bought 195, write an inequality expressing this purchase.

There are two different equalities described in the problem. These need to be separated out to create two different equations. The first equality tells how many boxes of chocolate were purchased. This will be the number of small boxes and large boxes. The first equality is small boxes and large boxes = 8 boxes of chocolate. The second equality tells how many pieces of chocolate are purchased. This includes all pieces in the small boxes and all pieces in the large boxes. The second equality is pieces in small boxes and pieces in large boxes = combined total of 195 pieces.

ACTIVITY 3.A

9.Claudia and Nick design a model rocket. They run a test flight and find the height of the rocket in meters is given by the formula h = (-5t + 30) × (t + 2), where t is the flight time after fuel burn out in seconds. The height is h = 0 when the rocket hits the ground. How long into the flight does the rocket hit the ground?

ACTIVITY 3.B

Fill a large box with marbles and a small box with marbles. Ask if someone bought a total of some number, to write an inequality to describe the purchase.

Rearrange formulas

Math.CCSS.Math.Content.HSA-CED.A.4

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's Law V = IR to highlight resistance R.

Students should be able to match commonly encountered formulas to context in word problems as well as rearranging them to solve for whatever value they want. They also need to be able to translate word problems into equations, students also need to be able to identify when a common formula is needed for the given context. This assumes that students are already familiar with the relevant formulas from previous learning. Students who are not already familiar with the formulas need to be supported in understanding them.

Sometimes, students may be trying to figure out an equality or inequality and realize they are missing some information. Students need to be able to identify the formula that describes the relationship using clues. Once a formula is written, students can manipulate it however they want. The process for rearranging it is identical to the process of rearranging any equation. It involves simplifying expressions and solving equations, which students should already know how to do.

There are many possible examples of activities for this standard. Activity 4.A shows an example of identifying formulas.

The possible answers are:

(A) A = lw

(B) A = lh2

(C) V = lwh

(D) A = 2lw + 2lh + 2wh

The correct answer would be A, or “A=*lw.*” The Addams family's deck is a rectangle with the specifications 12 feet long and 8 feet wide. Area A may be calculated as lw, where l = length and w = width.

Activity 4.B provides an example where students have chosen a formula and now have to apply it to the given details.

The possible answers for this activity could be:

(A) 81.7 ft3

(B) 31.4 ft3

ACTIVITY 4.A

1.The Addams family builds a rectangular deck outside their creepy mansion that is 12 feet long and 8 feet wide. Which of these formulas could be utilized to calculate the area of the deck?

ACTIVITY 2.B

The Addams family also wants to build a hot tub with a radius of 3 ft. The depth is 4 ft in the middle. All around the edge, a step extends 1 ft into the tub at a depth of 2 ft. What is the volume of the hot tub in ft3?

(C) 113 ft3

(D) 452 ft3

Since the Addams family's hot tub can be thought of as two cylinders, we can substitute the values for each one's radius and height. The first cylinder, from the surface of the hot tub down to the first step, has a radius of 3 ft and a height of 2ft. That gives V1 = π × (3 ft)2 × 2 ft for the first cylinder. The second cylinder has a radius of 3 ft – 1 ft = 2 ft and a height of 4 ft – 2 ft = 2 ft. If we substitute these values, we bet V2 = π × (2 ft)2 × 2 ft. Adding these two volumes gives V = 26π ≈ 81.7 ft3.

BENCHMARK ASSESSMENT

When assessing any common core standard it is important for us not to assess the technique, but the completion of the standard. Although many times the technique will be part of the standard and the only way for it to be met, we need to ensure its part of the standard if the technique is being assessed.

Therefore, for this cluster, any of the provided activities or derivation of them, will suffice as an assessment once combined into a test or quiz form. Students being able to hit the points of each standard will be assessed by completing and explaining the activities provided.

Also, some of the information necessary to complete the standard has been retained or remembered from 8^{th} grade Algebra or other previous lessons.