

Lesson Title: Proving all circles are similar

Unit Title: CIRCLES

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Subject, Grade Level, and Date: Geometry, 9th and 10th grade, 2/4/14

Placement of Lesson in Sequence

Introduction to CCSM cluster for circles.

Central Focus and Essential Questions

The central focus will be:

- The standard formula for a circle (review)
- Transformations on a circle/ translations of a circle
- Proving similarity of circles

Some questions to ask:

- Hypothetically speaking: are all circles similar to each other?
- How can I compare 2 circles?
- What sort of differences are there between 2 circles?
- What is the basic formula for a circle we know? ($x^2+y^2=1$)
 - What can I do to this to make the circle bigger? Smaller?
 - Move it left/right? Up/down?

Content Standards

CCSS.Math.Content.HSG-C.A.1

Learning Outcomes	Assessment
Students will be able to prove that all circles are similar through transformations.	Students must be able to prove that all circles are similar. Thus, they will be assessed on whether or not they can complete the provided problems. The worksheet will be graded out of 16 points, where each problem is worth 2 points.

Learning Targets	Student Voice
I can perform transformations on circles. I can prove 2 circles are similar to each other by using transformations and translations on the circles.	Students will be able to perform transformations on circles and be able to prove all circles are similar by performing transformations and translations until the circles are the same circle.

Prior Content Knowledge and Pre-Assessment

Students must know transformations on equations to be able to apply those transformations on the equation of a circle.

Academic Language Demands		
Vocabulary & Symbols	Language Functions	Precision, Syntax & Discourse
<ul style="list-style-type: none">• Transformation• Radius• Translation• Scale	Students must be able use the vocabulary to prove the similarity of 2 circles to successfully complete the standard.	Mathematical Precision: Students will need to know various methods to proving similarity. They will need to show

<ul style="list-style-type: none"> • center • Proof • Similarity • Dilation • Mapping 		this algebraically and geometrically.
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Language Target	Language Support	Assessment of Language Target
Using the vocabulary accurately to prove the similarity of two circles. Ex. "These two circles are similar because circle D has a dilation with a scale factor of 2"	We will be reviewing the terminology through the lesson so students can remember what all of the different vocabulary means.	Students will be writing down how they proved circles were similar on the worksheet. The assessment of language target being met will be part of the 2 point grade given for completion of the problem.

Lesson Rationale (Connection to previous instruction and Objective Standards)

This is the beginning to the Circles cluster in CCSM. Students will need to be comfortable with circles to continue forth with working with tangent lines, arcs, cords, etc.

Differentiation, Cultural Responsiveness and/or Accommodation for Individual Differences

Much of the work will be done individually. There will be a short lesson at the beginning of the class hour to review terminology and to begin a discussion on circles. The lesson will be using the document cam allowing all of the students to easily see the work being done and are required to take notes to see and write the lesson all the while hearing as the examples are being read aloud. Students who need to see the material again, can visit <http://www.youtube.com/watch?v=huGnhLIMUy4> to review.

Materials – Instructional and Technological Needs (attach worksheets used)

Overhead/document cam, graphing paper, compass.

Problems:

Show the two given circles are similar by stating the necessary transformations from C to D.

1. C: center (2, 3) radius 5; D: center (–1, 4) radius 10.
2. C: center (0, –3) radius 2; D: center (–2, 5) radius 6.
3. C: center (–2, 8) radius 4; D: center (0, 4) radius 9.
4. C: center (2, 8) radius 5; D: center (–2, 4) radius 1.

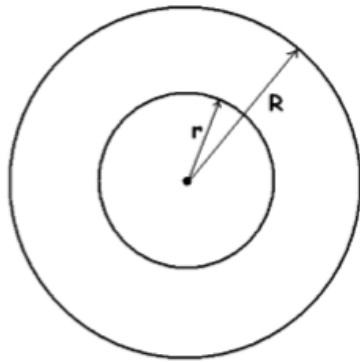
5. C: center (12, 32) radius 15; D: center (−1, 4) radius 10.

6. C: center (2, 0) radius 7; D: center (−1, 0) radius 4.

7. C: $(x+3)^2 + (y-2)^2 = 9$; D: $(x-1)^2 + (y-5)^2 = 25$

8. C: $(x-3)^2 + (y-4)^2 = 49$; D: $(x+1)^2 + y^2 = 1$

Teaching & Instructional Activities			
Time	Teacher Activity	Student Activity	Purpose
5min	Lead discussion <ul style="list-style-type: none"> Hypothetically speaking: are all circles similar to each other? How can I compare 2 circles? What are the differences between 2 circles? 	Discuss circles	Engage
20min	Lesson: Draw axis, draw a unit circle at origin and another circle shifted or dilated (or both) and discuss: <ul style="list-style-type: none"> What is the basic formula for a circle we know? ($x^2+y^2=1$) <ul style="list-style-type: none"> What can I do to this to make the circle bigger? Smaller? Move it left/right? Up/down? Label the circles, discuss that Circle A is about the origin so you would need to perform certain actions to make it look like circle B. Theorem: Any two circles are similar. Proof: Given a circle of radius r and a second circle of radius R, perform a translation so that their centers coincide.	Taking notes/asking question	Learn



A dilation from the common center of the circles with scale factor $k = R/r$ takes the points of one circle and maps them onto the second. Thus we have mapped one circle onto the other via a translation and a dilation. The circles are similar. \square

Example 1 Show that circle C with center $(-1, 2)$ and radius 3 is similar to circle D with center $(3, 4)$ radius 5.

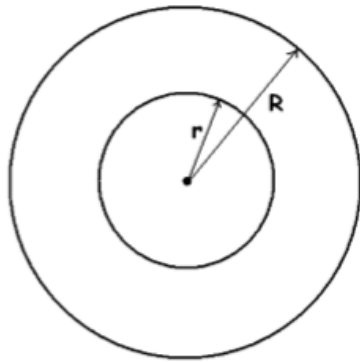
To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center 4 units to the right and two units up. To enlarge circle C to the same radius as D, the enlargement ratio is the quotient of the radii: $5/3$

Example 2 Show that circle C with center $(0, 2)$ and radius 6 is similar to circle D with center $(0, -6)$ radius 2.

To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center eight units down. To reduce circle C to the same radius as D, the reduction ratio is the quotient of the radii: $2/6 = 1/3$.

ANSWERS TO WORKSHEET: (translation; dilation)

1. left 3, up 1; 2
2. left 2, up 8; 3
3. right 2, down 4; $9/4$
4. left 4, down 4; $1/5$

	 <p>A dilation from the common center of the circles with scale factor $k = R/r$ takes the points of one circle and maps them onto the second. Thus we have mapped one circle onto the other via a translation and a dilation. The circles are similar. \square</p> <p>Example 1 Show that circle C with center $(-1, 2)$ and radius 3 is similar to circle D with center $(3, 4)$ radius 5.</p> <p>To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center 4 units to the right and two units up. To enlarge circle C to the same radius as D, the enlargement ratio is the quotient of the radii: $5/3$</p> <p>Example 2 Show that circle C with center $(0, 2)$ and radius 6 is similar to circle D with center $(0, -6)$ radius 2.</p> <p>To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center eight units down. To reduce circle C to the same radius as D, the reduction ratio is the quotient of the radii: $2/6 = 1/3$.</p>		
25min	Answering questions	Worksheet	Practice

5. left 13, down 28; $10/15 = 2/3$

6. left 3; $4/7$

7. right 2, up 3; $5/3$

8. left 4, down 4; $1/7$