Teaching Circles in High School Geometry According to the Common Core Standards

For 9th and 10th grade using "*Basic Geometry*" by Ray Jurgensen and Richard G. brown

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January 27, 2014

Understand and Apply Theorems About Circles

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Preface

This is a collection of methods for teaching of the geometry standards in the Common Core State Standards (CCSS) for high school. The target demographic for this learning progression is a school in Eastern Washington. The Geometry classroom will be 25-30 students varying from 9th through 10th grade. There are students who have special needs, thus lessons must allow for accommodations and modifications.

The textbook for this progression will be "Basic Geometry" by Ray C. Jurgensen and Richard G. Brown, 1988. We will primarily focus on Chapter 9, and students will be expected to have successfully completed all previous common core standards. Ch. 9.1 will introduce terminology regarding circles: Radius, Chord, Diameter, Secant, Tangent, point of tangency. The book chosen for this cluster or standards does not go in the order that the standards are listed, but we will ensure to cover all 4 standards.

Students will need to know how to use a compass and straight edge as well as know the general format for proofs prior to this cluster. The cluster would take approximately 2 weeks to cover. The cluster is going to be a part of the High School Geometry cluster under circles called "Understand and apply theorems about circles." The common core standards for 8th grade Geometry has left students off in a great place to pick up for high school geometry. They have learned:

- CCSS.Math.Content.8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:
- CCSS.Math.Content.8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- CCSS.Math.Content.8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Every day will begin with an entry task. Students will begin the lesson by exploring the prior or day's learning objective.

Proving All Circles are Similar

CCSS.Math.Content.HSG-C.A.1

Prove that all circles are similar.

By this point in a student's education, they have learned about proofs, postulates, and some general information on circles, such as: radii, circumference, translations, dilations, etc.

The proving of similar circles for CCSS.Math.Content.HSG-C.A.1 students must know how to translate images on a Cartesian plane. Unfortunately, our book does not have a good section for this standard and we must be a little more creative.

In general, two figures are similar if there is a set of transformations that will move one figure exactly covering the other. To prove any two circles are similar, only a translation (slide) and dilation (enlargement or reduction) are necessary. This can always be done by using the differences in the center coordinates to determine the translation and determining the quotient of the radii for the dilation.

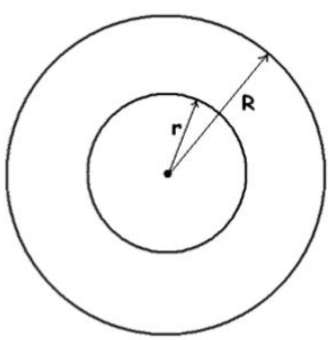
Therefore an acceptable form of proof for such a standard would be to prove that a circle, circle A, is similar to a second given circle, circle B, by completing translations and dilations on circle B to make their radii match.

We want to show the students a general proof for the theorem. The proof for Activity 1.A would be as follows:

Activity 1. A

Theorem: Any two circles are similar

PROOF: Given a circle of radius r and a second circle of radius R, perform a translation so that their centers coincide.



A dilation from the common center of the circles with scale factor

$k = \frac{R}{r}$ takes the points of one circle and maps them onto the second. Thus we have mapped one circle onto the other via a translation and a dilation. The circles are similar. □

After learning the theorem, students must be able to apply the rationale to multiple scenarios. Thus, providing them with practice allows students the opportunity to see multiple situations where this may be applicable. Activity 1.B is one of such examples. A list is provided below this activity with other possible examples.

To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center 4 units to the right and two units up. To enlarge circle C to the same radius as D, the enlargement ratio is the quotient of the radii: $\frac{5}{3}$.

Upon proving through assessment that a student has understood and can prove similarity of circles, then he or she is ready to move to the next standard. For those students who need a little more prompting, there are videos and manipulatives online that can be accessed for students to practice this standard.

RECOMMENDED EXERCISES: P.347 1-8, p. 348 11-31 odds only

Activity 1. B

Show that the circle C with center at (-1,2) and radius 3 is similar to circle D with center at (3,4) radius 5.

Show the two given circles are similar by stating the necessary transformations from C to D.

- C: center (2, 3) radius 5; D: center (–1, 4) radius 10.
- C: center (0, –3) radius 2; D: center (–2, 5) radius 6.
- C: $(x+3)^2 + (y-2)^2 = 9$; D: $(x-1)^2 + (y-5)^2 = 25$

Ch. 9.2 Explore the Concept of Tangent Lines

CCSS.Math.Content.HSG-C.A.4

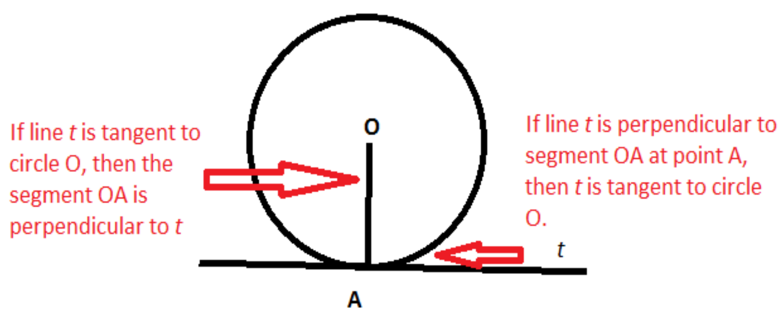
Construct a tangent line from a point outside a given circle to the circle.

The construction of the tangent line to a circle from a point outside of the circle requires specific knowledge. Therefore, we must begin by exploring tangents as we do in chapter 9.2. We must let the students have fun with this and they will need to be comfortable with a compass and straight edge for this lesson.

Using the EXPLORATIONS section, show students what makes a tangent line. Have students come up to either a document cam or board to complete the tangent line or radii. Which will lead to Theorems 1 &2.

Theorem 1

A radius is drawn to a point of tangency is perpendicular to the tangent



Theorem 2

If a line lies in the plane of a circle and is perpendicular to a radius at its outer endpoint, the line is tangent to the circle.

Review the terminology regarding internal/external tangencies. Allowing for students to do a lot of the critical thinking.

ACTIVITY 2.A

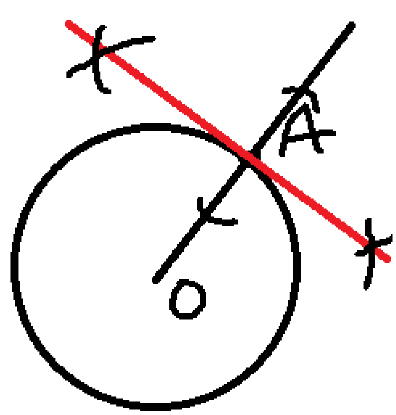
Construction (10 in book):

Given: Point A on circle O

Construct: a tangent to circle O at point A.

Students at their desk should follow along with the ACTIVITY 2 while at their desks.

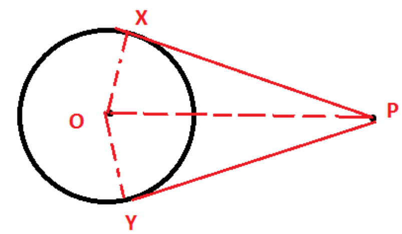
Have the students draw the given, while telling us the next steps to complete the tangent.

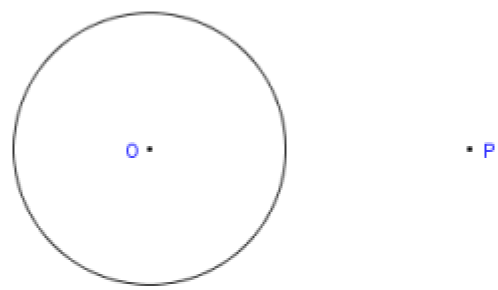
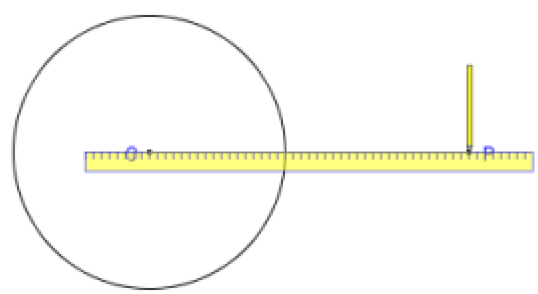
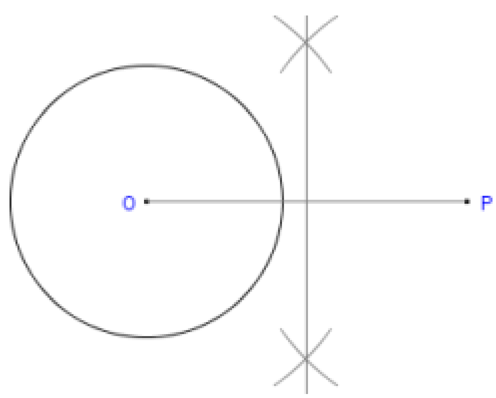


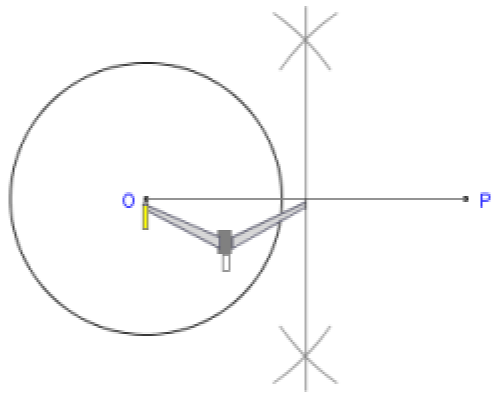
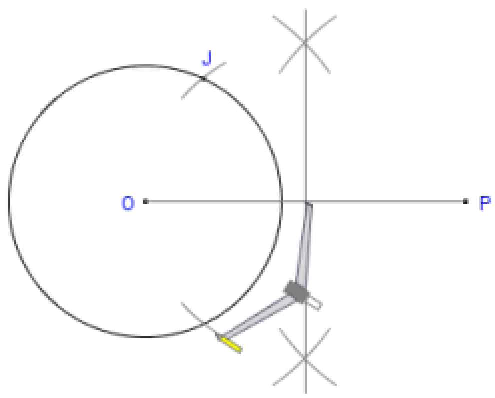
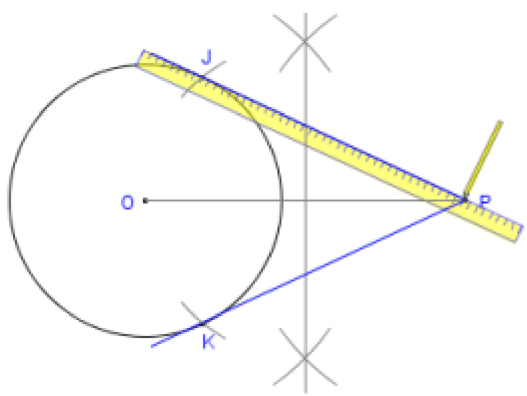
Have the students get into groups of 2 or 3 and have them try to construct a tangent from a point, P, outside the circle using their straight edge and compass. There will be two tangent lines, PK and PJ. Complete classroom practice problems on page 352.

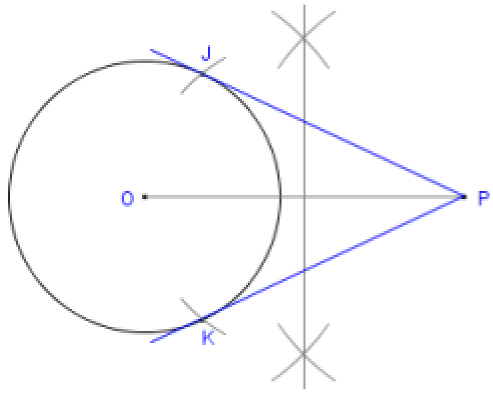
ACTIVITY 2.B

Draw a circle O, choose a point, P, outside the circle. Draw tangents, PX and PY, to the circle.



After doing this	Your work should look like this
We start with a given circle with center O, and a point P outside the circle.	
1. Draw a straight line between the center O of the given circle and the given point P.	
2. Find the midpoint of this line by constructing the line's perpendicular bisector.	

After doing this	Your work should look like this
3. Place the compasses on the midpoint just constructed, and set its width to the center O of the circle.	
4. Without changing the width, draw an arc across the circle in the two possible places. These are the contact points J, K for the tangents.	
5. Draw the two tangent lines from P through J and K.	

After doing this	Your work should look like this
6. Done. The two lines just drawn are tangential to the given circle and pass through P.	

Question 8: What appears to be true about PX and PY? (in our diagram PJ and PK)

PX=PY

Using the given diagram from activity 2.B complete Classroom practice from page 352, questions 8 through 14. This will give students comprehension as to why the two lines are accepted to be tangent lines.

RECOMMENDED EXERCISES: Pg.353 13-18

9.3-9.5 Chords and Circumscribing Polygons and Inscribing Angles

CCSS.Math.Content.HSG-C.A.2

Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

CCSS.Math.Content.HSG-C.A.3

Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

When it comes to circles, students often think that they don't have to worry about angles anymore. After all, a circle is a curve. Before students can identify and describe the various angles in a circle, they should be familiar with what these angles are. Students should also be familiar with the concept of arc measurement and how it relates to the measure of the different kinds of angles.

Students should know that a central angle is formed by two radii (where the vertex of the angle is the center of the circle), an inscribed angle is an angle formed by two chords (where the vertex of the angle is some point on the circle), and a circumscribed angle has a vertex outside the circle and sides that intersect with the circle.

After students have mastered these angles and arc measures, talk about tangents and secants of circles, lines that intersect a circle at one and two points, respectively. It's also important for students to know that a point on a circle can only have one tangent and that tangents are always perpendicular to the radius of a circle.

Some examples of problems to ensure we are assessing the standard, I have attached some sample questions:

1. What is the measure of a central angle with an arc measure of 45°?

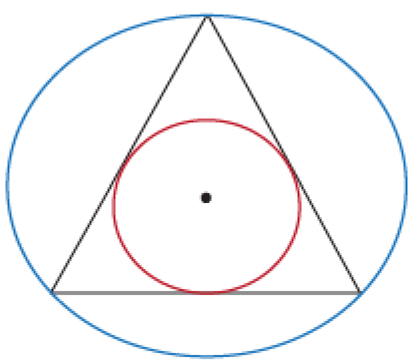
2. What is the measure of an inscribed angle with an arc measurement of 150°?

3. What is the measure of an inscribed angle with an arc measurement of 70°?

4. What is the measure of the arc with an inscribed angle is 18°?

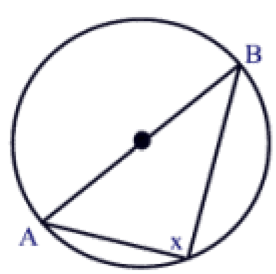
Answers are: 45 degrees, 75 degrees, 35 degrees, 36 degrees.

Students should know that an inscribed circle is the largest circle that can fit on the inside of a triangle, with the three sides of the triangle tangent to the circle. A circumscribed circle is one that contains the three vertices of the triangle. Students should know the difference between and be able to construct both of these circles.



Students should understand that circles have one defined center, but triangles have four different centers: the incenter, the circumcenter, the centroid, and the orthocenter. Students should know the definitions of these centers and how each one differs from the others. But which one is the real center of the triangle!?

A student should be able to draw any triangle by plotting three points on a circle. They can take this one point further, plot four points on a circle, connect them, and make a cyclic quadrilateral.



Students should be able to prove theorems about cyclic quadrilaterals, the most important of which is that opposite angles in a cyclic quadrilateral add to 180°. Or they can say that opposite angles in a cyclic quadrilateral are supplementary. See figure 3 as a few examples of this type of work.

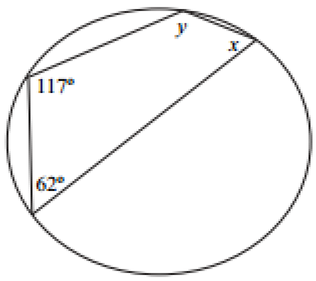


Figure 1

1. How would you construct a circle that circumscribes a given triangle?

2. Why is the center of the circle that circumscribes a triangle at the intersection of the perpendicular bisectors?

3. Which theorem is needed to prove that the opposite angles in a cyclic quadrilateral are supplementary?

   Refer to the Figure 1 for questions 4 and 5.

4. Determine the measure of $\angle x$.

5. Determine the measure of $\angle y$.

For question 1, students would have to state the need for perpendicular bisectors of each side of a triangle. Find the point of intersection (which will be the center we want) and draw a circle through one of the vertices. The circle should pass through the other 2 vertices as well.

Question 2 would require an understanding that a perpendicular bisector of a segment is equidistant from the endpoints. Therefore every point on each side's perpendicular bisector is equidistant from the vertices, which means that the intersection of each perpendicular bisector will be equidistant from the vertices of the triangle. Resulting in the center of the circle that circumscribes it.

The 3rd question requires some memory of our theorems, which will be in this case, the Inscribed Angle Theorem.

Students will need to think about supplementary angles, sum of angles to find the answers for 4 and 5. Which are 63 and 118 degrees, respectively.

BENCHMARK ASSESSMENT

When assessing any common core standard it is important for us not to assess the technique, but the completion of the standard. Although many times the technique will be part of the standard and the only way for it to be met, we need to ensure its part of the standard if the technique is being assessed.

Therefore, for this cluster, any of the provided activities or derivation of them, will suffice as an assessment once combined into a test or quiz form. Students being able to hit the points of each standard will be assessed by completing and explaining the activities provided.

Also, some of the information necessary to complete the standard has been retained or remembered from 8^{th} grade Geometry or other previous lessons.