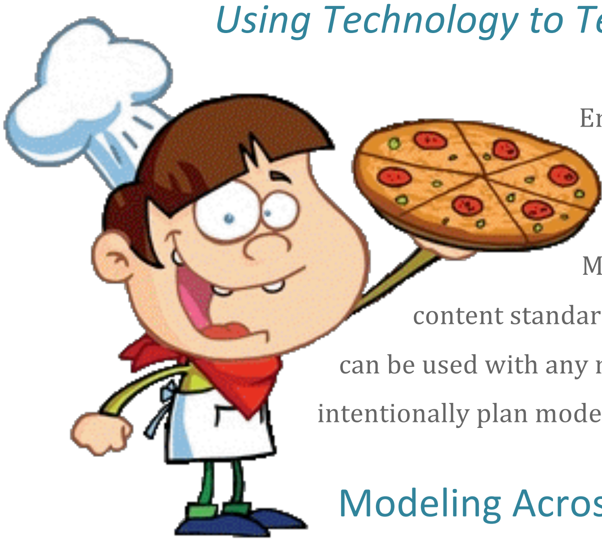
Pizza, Pizza, Pizza

Using Technology to Teach Mathematical Modeling



Embedding mathematical modeling into the mathematics curriculum is a challenge. Mathematical modeling is a standard for Mathematical Practice and connected to the content standards by a star symbol (*). Even though modeling can be used with any mathematical problem, teachers need to intentionally plan modeling lessons for CCSS-Math with the star symbol.

Modeling Across the Curriculum

Ensuring students have opportunities to relate and connect key math concepts to everyday situations is very challenging. When planning a modeling activity teacher must ask himself or herself; "How am I scaffolding modeling instruction to support learning for all students?" Teachers must scaffold lessons by leveraging student experiences from in and out of the math classroom. A good math model lesson starts with an everyday problem or situation that all the students understand and can talk about. The teacher then must scaffold the lesson (implement learning structures) to highlight the important math objects and plan supports for students who struggle at see situation from a mathematical perspective. Math objects are aspects of the everyday situation that can be represented mathematically. For example, modeling how much faster one car is than another requires students to identify some combination of the following mathematical objects, speed, time, or distance.

Modeling Activity

Pizza, Pizza, Pizza is a modeling activity that aligns with standard F-FB1 (students will build a function relating two quantities). The students in this example have built some linear functions and now it is time for them to build a quadratic function. Since standard F-FB1 is a star standard, this activity will use mathematical modeling as the primary Mathematical Practice. Eating pizza is something all students have experienced, so a pizza problem should engage the students in conversation. A worksheet will be used to scaffold the modeling process, by guiding the students' modeling steps and elicit their responses.

Central Focus of this lesson:

These students have worked with linear functions as a relationship between two quantities with equal differences over equal intervals. Most recently the students have studied the properties of quadratic functions in graphical, numeric, and equation form. In this activity students will solve a mathematical modeling activity that uses the their knowledge of the area of a circle sector to build a linear and quadratic function. Students will be presented with the question, "Which is more pizza, two slices from a 9" pizza or one slice from an 18" pizza?"; then they will be asked to model the area of a slice of pizza as it relates to doubling the sector angle of a circle or doubling the radius of a circle. Finally, students will be asked to generalize this problem to equations related to the area of a circle sector.

CCSS-Math:

- F-BF1 Build a function that models a relationship between two quantities.
- MP1 Making sense of problems and persevere in solving them.
- MP4 Modeling with mathematics.
- MP5 Use appropriate tools strategically
- MP7 Looking for and making use of structures.

Scaffolding for Supporting Students

A worksheet and combination of small groups and whole class instruction were designed to guide and support students in building a mathematical model. Giving students with varying needs the support to build a model to solve the problem is very important. The modeling steps from the worksheet and the planned supports are explained below.

1. **Support students in understanding the everyday problem from a mathematical point of view.**

- Make assumptions
- Identify related facts
- Identify needed information
- Identify important quantities and explain how they are related
- Organize information into equations, tables, charts, graphs, and diagrams

Students worked in small groups to complete the understanding the problem section of the worksheet. After the groups discussed the pizza situation for about 5 minutes the teacher had the students vote whether they would order two slices of the 9" pizza or one slice of the 18" pizza. It is important to have students predict their solutions so that both the teacher and students can identify new insights.

2. Support students in building a model to solve the everyday problem.

- Look for math structures that relate the quantities to build a model
- Realize when revision to model need to be made
- Use the model to solve the problem

The teacher asked students to share their assumptions and knowledge about the pizza problem. The teacher used the student answers, to ask others students, how this information could be used to build a geometric model. The teacher used the students' ideas about a geometric model to create the Geogebra model. Finally, students were prompted to use the Geogebra model to solve the pizza problem.

3. **Support students in evaluating and improving their model.**

- Check to make sure the solution makes sense
- Make improvements to the model or strategy
- Generalize the model to bigger math ideas or concepts
- In this example students are asked to generalize their slice pizza

Students worked individually to record and explain their solution to the pizza problem. They also generalize the pizza problem by writing equations related to the area of a circle sector for each pizza solution option.

Using Geogebra to Model a Slice of Pizza

A drawing of a pizza or actual pizza pans could be used to represent a doubling pizza, but the teacher wanted students to visualize what happens to the area of a circle sector when the sector angle is changed or the radius is changed. With a drawing or pizza pans students would be required to use the formula for the area of a circle sector to collect data for the solution. Using Geogebra allows students to visualize and collect data for this pizza problem without calculating the areas of circle sectors. When teaching students how to build models it is important to suppress some mathematical options, in order to highlight others. Geogebra allowed the teacher to highlight changes in the area of the circle sector as the sector angle and circle radius were changed independently. Using student information and suggestions to create a computer models can be tricky. For this reason the teacher planned to use one of two possible Geogebra constructions for the pizza model. Each construction has pros and cons but knowing two possible constructions allowed the teacher to use student input in the building of the Geogebra pizza model.

Model pizza slice with a <u>single</u> circle with center through point.

- If a student states to represent a pizza with a circle, the teacher can use the "circle with center through point" drawing tool to construct a circle with a varying radius.
- If a student states that the slice of pizza must have a set angle, the teacher can construct a circle sector (construct two points on the circle, construct two line segments between the points on the circle and center of the circle, use the circle sector tool to create a shaded sector between the two points on the circle).
- When students identify needed measurement, the teacher can measure one of the radii, the angle of the circle sector, and the area of the circle sector.

Figure 1 represents one eighth of a 9" pizza and figure 2 represents one four of a 9" pizza (doubling the sector angle or two pieces of the 9" pizza). Students could see that the pizza area doubled when the sector angle was doubled, while the radius was unchanged. Figure 3 represents one eighth of an 18" pizza (doubling the radius while keeping the angle of the circle sector the same).

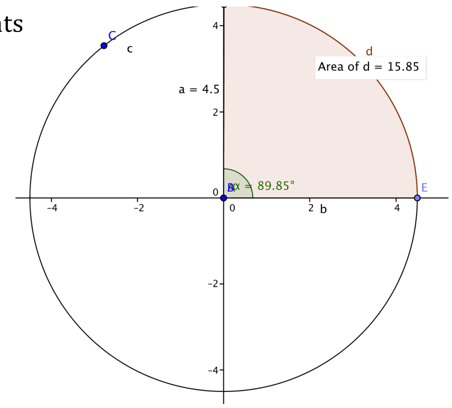


Figure 2

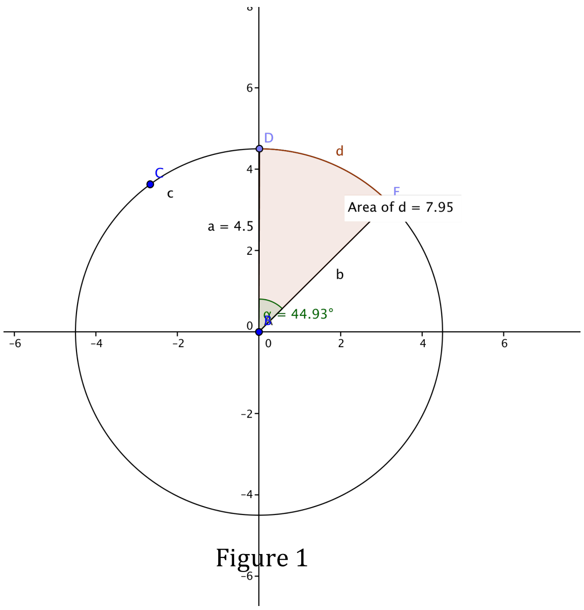


Figure 1

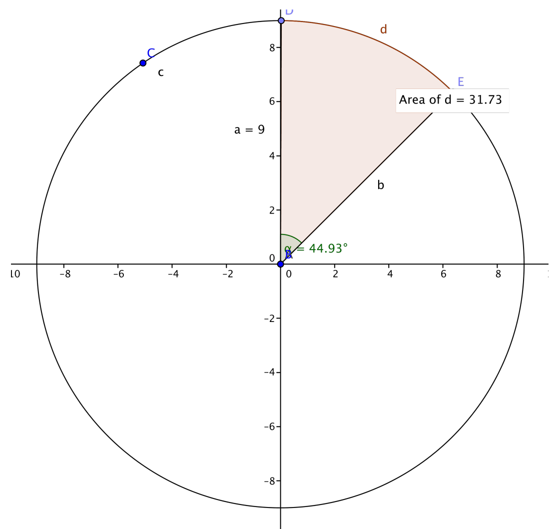


Figure 3

Model pizza slice with <u>two</u> circles with center and radius.

- If a student states that we need to construct two pizzas, the teacher can use the "circle with center and radius" drawing tool to construct two circles with their centers at the origin and radii 4.5 and 9 respectfully.
- If a student insists that the pizzas are cut into a select number of equal size slices, the teacher can construct diameters to divide the pizza into fourth, sixth, or eights (notice to cut the pizza into eighth, the teacher will need to construct chords and perpendicularly bisect the chords to create the diagonal diameters).

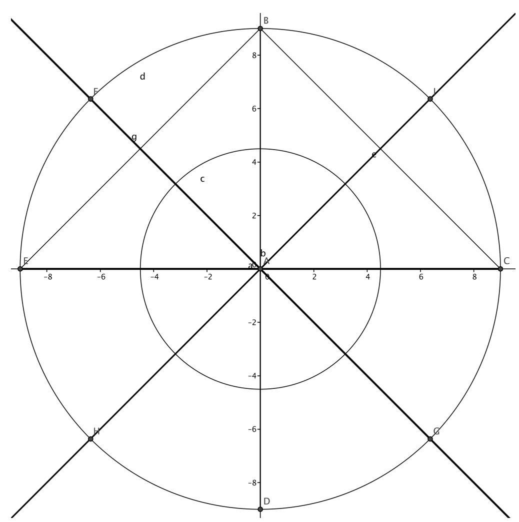


Figure 4

- When students identify needed measurement, the teacher can measure the area of the circle sector for one slice of the 9" pizza, two slices of the 9" pizza, and one slice of the 18" pizza.

Figure 4 represents the construction of two concentric circles with diameters of 9 and 18 that are cut by four diameters to create 8 equal circle sectors. Figure 5 represents the area of one eighth of a 9" pizza, one four of a 9" pizza (doubling the sector angle or two slices), and one eighth of the 18" pizza. Students can see that two 9" slices doubles the pizza area, while one 18" slice is four times the area of one 9" slice.

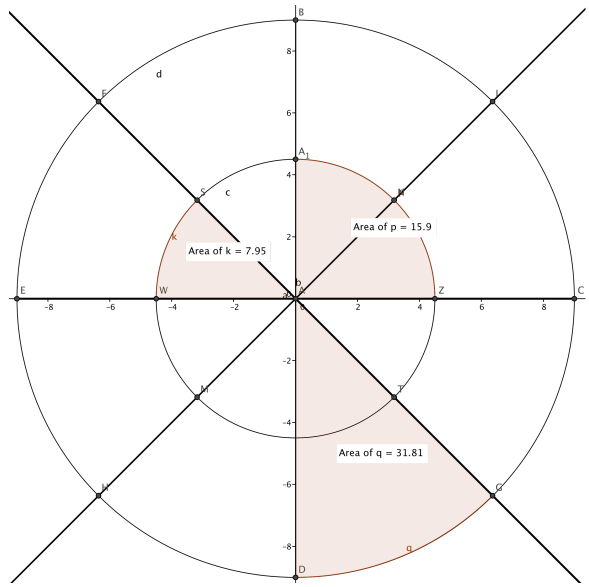


Figure 5

Pros and Cons of each Geogebra Construction

The first Geogebra construction is easier and quicker to make but since most students will not understand that the radius and sector angle can dynamically change when using Geogebra, the modeling process will become more teacher lead.

The second Geogebra construction has many steps but the model looks like a pizza cut into eight slices. This model also visually represent all the needed information to answer the pizza problem at once, but it is a static model that does not visually reveal how the sector angle or radius can be used as parameter in the equation of a circle sector.

Encourage Discussion and Use of the Geogebra Model

This function building and modeling building activity never explicitly used the formula for the area of a circle sector, but some knowledge of this formula is need to express the pizza problem in function notation. In this problem there are two independent variables and in each case one of the variables is made a constant while the other is manipulated. This makes for great conversation about how to express functions. For example some students express the changing the sector angle as doubling the numbers of pizza slices by writing $A = \frac{1}{8}\pi r^2$ changes to $A = \frac{2}{8}\pi r^2$. We know what these students mean but when teaching function building, the dependent and independent variables must be expressed in function notation. The teacher responded, "How can you represent a changing number of slices in function notations?” The teacher was looking for an equation like $A(\theta) = (\frac{\theta}{360})\pi r^2$ but most students wrote $A(x) = \frac{x}{8}\pi(4.5)^2$ or even $A(x) = \frac{x}{8}63.6$. These students were thinking about a pizza cut into eight slices and are correct since they were able to use function notation to show the linearly relationship. When representing the change in the diameter of the pizza most students wrote $A = \frac{1}{8}\pi(4.5)^2$ changed to $A = \frac{1}{8}\pi(9)^2$. Again function notation requires that students identify the dependent and independent variables and when prompted to write their equation in function notation, most students were able to write something similar to $A(r) = \frac{1}{8}\pi r^2$. The teacher was pleased that at least half of the students were able to write the two the pizza problem options in function notation with out too much prompting, but as with many modeling activities student often express complex ideas which lead to great conversations and difficult explanations. One student expressed the pizza problem with the one equation $A = \frac{a}{360}\pi(br)^2$, this student like the other students needed prompting to express her equation in function notation. After much discussion the following equation was suggested $A(a,b) = \frac{a}{360}\pi(br)^2$. This student did not see the solution to the pizza problem as two separate problems but as one problem. Because the problem was posed as a doubling problem and the Geogebra model was the single circle with a dynamic sector angle, she saw one equation with two parameters. Most of her classmates did not understand her function but it was a great conversation and she was empowered to express her mathematical understanding.

What Worked

This was a good modeling activity because the students liked talking about pizza. The student were able to use worksheet with out difficulty, which gave them guidance and the teacher lesson structure. The worksheet was the primary form of assessment. Geogebra was a powerful tool for building a visual model, but because the students had not seen the computer program before, many students did not understand the Geogebra model. The teacher plans to use Geogebra as a regular tool in this math course and like any tool the students should become more comfortable with the program as they become more skillful in its use.