

Overview

The excel Algebra class is based on upon grade 7 curriculum. Grades are received based on participation. Students are capable of grade level achievement they just need guided instruction and supporting materials such as visual aids. They also need hands on activities so they are able to see how the mathematics is applicable in real life to give them motivation to learn the curriculum. When learning about the ratio and proportional relationship CCSS.MATH.CONTENT.7.R.P. students expand upon their previous knowledge about ratios and develop an understanding of proportional relationships. Students use ratios to represent real world scenarios and compute unit rates in order to compare ratios with the same units. They use equations to represent and analyze proportional relationships. They also must be able to graphically represent proportional relationships and explain what each coordinate on the graph represents in terms of the scenario. Students will also have to solve multi-step ratio and percent problems. By the end of this unit students will be able to solve problems involving discounts, taxes, percentage increase and decrease, and use scale models. Throughout the learning progression there are multiple mathematical practices and visual representations to aid students with learning disabilities and different learning styles to help them achieve mathematical understanding regarding ratios and proportions.

Identify if two quantities are in a proportional relationship

Students will examine scenarios and determine if a proportional relationship is described based upon verbal context and algebraic methods.

7.RP.2a For example, a lead question may be: if there are 6 goldfish for every 11 fish in the lake. How many gold fish will there be if there is a total of 150 fish in the lake. This model is a real life biology scenario which interest high school students who are familiar with different types of animal. The biology model also motivates students to participate because it makes the mathematical procedures seem useful. This problem can be solved with the proportion $\frac{6}{11} = \frac{f}{150}$ because ratio of gold fish to total fish is consistent through out the whole lake.

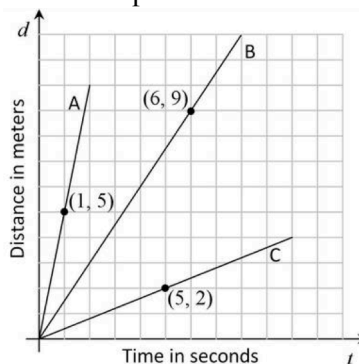
Students also identify if relationships are not proportional. For example, it takes 3 people 16 hours to build a tree house. How long will it take 5 people to build a tree house of the same size and design? We cannot solve this problem with the proportion $\frac{3}{16} = \frac{5}{H}$ because it is not the case that for every 3 people, 16 hours of work is needed. With $\frac{2}{3}$ more people the work will be completed in a shorter amount of hours. Most high school students built a tree house or played in their friend's tree house growing up so using this scenario puts proportions in a relatable perspective for them.

MP.1. Make sense of problems and persevere in solving them

7.RP.2a Decide whether two quantities are in a proportional relationship.

Carli's class built some solar-powered robots. They raced the robots in the parking lot of the school. The graphs below are all line segments that show the distance d , in meters, that each of three robots traveled after t seconds.

- b. Carli said that the ratio between the number of seconds each robot travels and the number of meters it has traveled is constant. Is she correct? Explain.



Solution: Carli is correct. Whenever the ratio between two quantities is constant, the graph of the relationship between them is a straight line through $(0,0)$. We can also say that for each robot, the relationship between the time and distance is a proportional relationship.

By distinguishing between proportional relationships and non-proportional relationships, students are making sense of the problems and persevere in solving them.^{MP.1}

The students will also identify proportional relationships through kinesthetic learning styles by using the kinesthetic M&M activity and the algebra method of cross products. Students will only be able to eat the M&M's after completing the activity, which will be incentive to them to stay focused and attentive. Every pair of students will receive a bag of M&M and given scenarios that requires them to identify if there exists a proportional relationship. For instance, there are 1 red M&M for every 3 M&M's and 3 blue M&M for every 9 M&M. To identify if these ratios are proportional the students will use a cross product method.

$$\begin{array}{l} \frac{1}{3} = \frac{3}{9} \\ 1 * 9 = 3 * 3 \\ 9 = 9 \end{array}$$

Therefore the ratios are proportional. If the cross products are not equal the students will have to identify the ratios as not proportional to correctly identify the relationship. Proceed by allowing students to use their creativity and develop their own proportional relationships using the M&Ms and record them.

A common mistake in setting up proportions is placing variables in incorrect locations. For example, the first problem of the two following statements is more difficult because the statement says the variables in reverse order of how you would set them up in a proportion.

If Jerry has 5 yellows M&M for every 3 brown M&M in his bag, then how many brown M&Ms would Jerry have when he has 25 yellow M&M's in his bag?

Compared to the straight forward statement:

If Jerry has 5 yellows M&M for every 3 brown M&M in his bag, then when Jerry has 25 yellow M&M's how many brown M&Ms would Jerry have?

Students with learning disorders such as dyslexia may especially find reversing the order more difficult so it is best to stick to the second statement when presenting the problem.

Then the students use their knowledge about proportional relationships in a graphical relationship. After being shown a series of visual aids of graphs representing proportional relationships.

Analyzing tables of the graph students will also learn that the coordinates increase or decrease at a constant rate just as linear functions do.

Students recognize that only graphs with lines that do go through the origin and tables that do have constant ratios, represent proportional relationships. For example, consider the benchmark assessment: robot problem. It represents the d , distance in meters that each of the three robots traveled after t , seconds. Carli correctly identified the ratio of seconds to distances traveled as constant by analyzing the graph and seeing that the shows straight lines through $(0,0)$ and if a table were made to list the coordinates the ratio would be constant. This provides students with the opportunity to identify proportional relationships graphically by visually seeing a linear function and algebraically by analyzing the constant ratio between the coordinates.

Equations for proportional relationships

As students develop skills regarding proportional relationships, they will compute unit rates ^{7.RP.1} to write equations of the form $y=cx$, where c is a constant of proportionality, e.g., a unit rate. ^{7.RP.2c}

They are able to identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

^{7.RP.2b} Students will connect the concept of unit rate and use that to identify the unit rate and constant of proportionality in the benchmark assessment.

The central problem of this activity :“A pizza with 8 slices evenly proportioned cost \$14.80, what is the cost of each slice of pizza?” This problem will be used as the real-world situation to find the unit rate, make a table and graph to represent the data, and as information for an equation to represent the proportional relationship. For example, 8 slices was worth \$14.80 so dividing 14.80 by 8 the students can find each slices to be \$1.85. Kinesthetic learners should have the opportunity to model with mathematics^{MP.4} by physically making the pizza and writing the price of each slice directly on their piece. Relating this information to unit rate definition students can write a unit ratio for one slice of pizza. This

MP.4. Model with mathematics

M.P.3. Construct viable arguments and critique the reasoning of others

7.RP.1. Compute unit rates associated with ratios of fractions

7.RP.2c. Represent proportional relationships by equations.

7.RP.2b They are able to identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Coffee costs \$18.96 for 3 pounds.

- What is the cost for one pound of coffee?
- At this store, the price for a pound of coffee is the same no matter how many pounds you buy. Let x be the number of pounds of coffee and y be the total cost of x pounds. Write an equation to represent the scenario.
- Draw a graph of the relationship between the number of pounds of coffee and the total cost.
- Where can you see the cost per pound of coffee in the graph? What is it?

Solutions

- You can find the cost for one pound of coffee by dividing the total cost by 3. Coffee costs \$6.32 per pound.
- $y=6.32x$
- We may graph the proportional relationship between the total cost and the number of pounds by plotting the line through the origin and $(3, 18.96)$.
- The cost of one pound, \$6.32, may be seen on the graph in two ways:

As the point $(1, 6.32)$

As the slope of the line: \$6.32 per pound

process would allow the students to develop an equation representing the cost of each slice which they could then plug values in to form a table in order to generate a graph. Students who struggle will be support their peers because they are working in groups. Visual learners should have the opportunity to model with mathematics by physically making the pizza and writing the price of each slice directly on their piece. To further assist students and visual learners the graph can be created in Geogebra to be used as an organized graphical representation. Students order pizzas and other take out food, so being able to solve this problem will be useful for them to figure out how to save money and get the better deal when ordering food. If some students do not like pizza you could give them a different type of food to peak all students interests and make it relatable to them and increase the diversity of curriculum in the classroom.

The coffee benchmark assessment has students do a similar mathematical process that the students had to perform in the pizza activity. Part d has the students perform the mathematical procedure in reverse by using the graph to generate an equation and identify the unit rate.

For their formative assessment the students will also have to verbally explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation. ^{2.RP.2d} For instance, using the coffee scenario. The students would have to explain that point (1,6.32) tells that 1 pound of coffee costs \$6.32. In their explanation is a good time for students to construct viable arguments and critique the reasoning of others^{MP.3} based on their peers' response to what different coordinates represent.

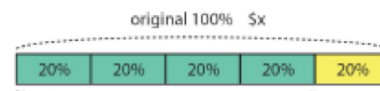
Multistep problems Students expand upon their knowledge of ratios and proportional relationships to solve multistep ratio and percent problems. ^{7.RP.3} For the multistep problems activity students will be asked to solve problems involving discounted and increased prices. In groups students will make posters presenting the problems and justifying to their peers how they found their solution.

Students solving problems involving percent increase or decrease have to pay attention to the referent whole. For example, consider the two problems below.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems

Shoes problem. If a pair of shoes cost \$40 and is advertised at 20% off the original price, what is the sales price?

Method 1



The whole bar represents the total cost of the shoes which is \$40. Each bar is then \$8. 20% off means that yellow bar is not included in the new price. Therefore the sale price is \$32.

Method 2

20% of the price.

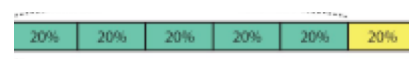
Of means multiple. Therefore, $.20 \times 40 = 8$.
And $40 - 8 = 32$

Method 3

$1 - .20 = .80$

80% of price means $.80 \times 40 = 32$.

Pants problem: At Peter's Pants Palace a pair of pants usually sells for \$33.00. But they are in high demand and the manager of the store plans to raise the price by 20%. What will the new price be after the increase?



After a 20% increase the price is 120% of the original price so the new price is 120% of \$33.

$$x = \frac{120}{100} \times 33 = 39.6$$

The new price after the increase is \$39.6

- a. If a pair of shoes costs \$40 and is advertised at 20% off the original price, what is the sales price?
- b. At Peter's Pants Palace a pair of pants usually sells for \$33.00. But they are in high demand and the manager of the store plans to raise the price by 20%. What will the new price be after the increase?

These two problems have different solutions because in the first problem 20% is 20% of the original amount, whereas in the second problem, the 20% is 20% of the smaller pre-increased amount. Take note that the distributive property is used in the mathematical procedure of decreasing and increasing a percent. For instance, in the first problem if x is the original price of the shoes, then the new price is $x - 20\% \cdot x$. By the distributive property the new price is represented by: $80\% \cdot x$.

$$(x - 20\%) \cdot x = 100\% \cdot (x - 20\%) \cdot x = (100\% - 20\%)x = 80\% \cdot x$$

Distributive property is not the only method to solve these two problems. Notice there is several methods at the right. This gives creative learners a way to check their solutions multiple ways and slow learners variety of mathematical procedures to choose from in hopes of making the problem more understandable.

At this age students are starting to buy their own clothes so they need to understand discounts and mark ups to be smart with their money and have an idea of what the new price is going to be. Giving multiple methods also provides students with more opportunities to understand the material with a process that might be easier for them to understand. For instance, with the students who are visual learners they can use method 1 or 3 because the sale price is visually broken up in segments. The struggling students could use method two since it is only a two step process with basic mathematical practices: multiply, add, and subtract, Percentages can also be used to make comparisons as well as analyze and figure out commissions by using the equation $\text{commission} = \text{rate} \cdot \text{total sales amount}$.

Lesson Title: Pizza

Unit Title: Ratios and Proportions

Teacher Candidate: Liz

Subject, Grade Level, and Date: Excel Algebra , Freshman-Juniors, and 2/4/2015

Placement of Lesson in Sequence

This is an excel Algebra class towards the middle of the year. We have just reviewed identifying whether two quantities are in proportional relationships, and identifying the constant of proportionality (unit rates) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. This modeling activity will use students' knowledge of ratios, unit rates, and proportions that represent the cost of pizza students will order.

Central Focus and Essential Questions

The problem, "A pizza with 8 slices evenly proportioned cost \$14.80, what is the cost of each slice of pizza?" Will be used as the real-world situation to find the unit rate, make a table and graph to represent the data, and as information for an equation to represent the proportional relationship. Each student will make their own pizza with 8 slices evenly proportioned. Students will need to remember unit rate definition to find the unit rate and write the unit rate on each slice of pizza. Using Geogebra students will be asked to make a table and graph to represent their data. Students will be formatively assessed throughout the lesson to provide evidence to provide feedback about whether students are ready or not for the benchmark assessment that will be given the following day testing the standards gone over in this lesson. Students order pizzas and other take out food, so being able

to solve this problem will be useful for them to figure out how to save money and get the better deal when ordering food. If some students do not like pizza you could give them a different type of food to peak all students interests and make it relatable to them and increase the diversity of curriculum in the classroom.

CCSS-Math (Standards)

7.RP.1 Compute unit rates associated with ratios of fractions

7.RP.2c. Represent proportional relationships by equations.

7.RP.2b They are able to identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.2d The students will also have to verbally explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation.

MP.4. Model with mathematics.

Learning Outcomes	Assessment
<p>Students will identify assumptions and variables needed to solve the pizza problem.</p> <p>Students will identify and explain geometric objects needed to model the pizza problem using Geogebra.</p> <p>Students will use the data from the model of the pizza to give and justify a unit rate for the pizza problem.</p> <p>Students will use each slice of pizza to create a table representing the cost of each slice.</p> <p>Students will use their knowledge representing proportional relationships by creating an equation that represents the pizza problem that they are able to graph on Geogebra.</p> <p>Students will be able to explain what a coordinate on the graph of a proportional relationship means in terms of pizza slices and cost.</p>	<p>The think pair share formative assessment technique will be used. The students will think about how to find a unit rate for the pizza problem individually and then share ideas to the class. The students will then pair up with a partner and discuss answers of unit rates and a possible equation for the scenario. The teacher will go from group to group to assure students are understanding the problems by making assumptions and defining their unit rates correctly on the slices of pizza.</p> <p>Peer assessment between groups will be used to check suggested proportions to be used to create the pizza model.</p> <p>The teacher will go from table to table to assure students are using the unit rates on their pizza to generate a correct table and graph on Geogebra. The teacher asks different students to present their graphs for generalization of the unit rate and an equation that represents their solution. The students will also be asked to verbally explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation.</p> <p>Summative assessment will be the coffee benchmark assessment they complete the following class period.</p>

Learning Targets	Student Voice
<p>I will answer all prompts on the worksheet to:</p> <ol style="list-style-type: none"> 1. Solve the pizza problem; 2. Assist in creating a Geogebra graph to solve the pizza problem; and 3. Generalize the pizza problem to 	<p>The teacher will use the worksheet to communicate the learning target by asking the student to explain how they will use the cut out pizza slices to assist them to solve the pizza problem through the modeling process.</p>

represent the solution in equation form.	
--	--

Prior Content Knowledge and Pre-Assessment

Students have been working with linear functions and a linear modeling. Students have also just reviewed identifying whether two quantities are in proportional relationships, and identifying the constant of proportionality (unit rates) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Furthermore, students already know how to form single variable equations.

Academic Language Demands		
Vocabulary & Symbols	Language Functions	Precision, Syntax & Discourse
<ul style="list-style-type: none"> Identify variables in the pizza related to unit rate. Define ratio, proportional relationship 	<ul style="list-style-type: none"> Students will be able to discuss that since the slices are evenly proportioned you are able to use the total cost and total slices of pieces to find a unit rate that can be connected to creating an equation that can be modeled on Geogebra 	<p>Mathematical Precision: Students will accurately set up proportional relationship and correctly make a table representing the data.</p> <p>Syntax:</p> <ul style="list-style-type: none"> Identify the variables that are needed to write a proportion to calculate the unit rate Identify the variables in the equation representing the total cost of pizza in the Geogebra model for the proportional relationship <p>Discourse: Discuss how to accurately define ratio and proportional relationship and what those mean in terms of the pizza scenario.</p>

Language Target	Language Support	Assessment of Language Target
I will be able to identify variables in the pizza problem related to finding the unit rate in a proportional relationship.	Introduction whole group, review vocabulary related to ratios, unit rate, and proportional relationship. Small groups identify variables related to unit rate in the pizza problem needed to set up a proportion and report out to whole group for purpose of creating an equation. Teacher will repeat back student responses from small groups with correct syntax for writing and speaking of proportional	Formatively the teacher will give feedback by correctly modeling the verbal and written syntax for the unit rates and proportional relationships. The teacher will also walk around analyzing the students' tables and graphs to ensure that the students are accurately representing the proportional relationship with the correct unit rate. Summatively the teacher will use a rubric to assess the

	relationships. Students will discuss in small groups and writing individually on worksheets their solution to the pizza problem and generalization of the problem using an equation. Finally students will individual graph their solution to model the scenario on Geogebra.	benchmark assessment given to the students the following day. The teacher will assess the students' written responses to their solution to the coffee problem and generalization of the problem using an equation.
--	---	--

Lesson Rationale (Connection to previous instruction and Objective Standards)

This lesson is a next step for the students to expand upon their knowledge of ratios and proportional relationships to solve multistep ratio and percent problems. When pressed with more challenging problems students will be able to think back to this lesson and use visual representations and models to help assist them to find key information such as unit rates that are needed to formulate equations involving proportional relationships.

Differentiation, Cultural Responsiveness and/or Accommodation for Individual Differences

Students who struggle will be support their peers because they are working in groups. The program Geogebra will help student visualize the proportional relationship and related a linear equation that goes through an origin. Visual learners should have the opportunity to model with mathematics by physically making the pizza and writing the price of each slice directly on their piece. Students that have a hard time communicating or have a speech impediment may need more people in their group since they are not able to as easily share their ideas. If there is a few groups of three the students struggling with communication can do more listening and feel less pressure of the activity. Also ELL students that have a hard time speaking English fluently can be paired with a fluent bilingual student who can speak the same language or is good at interpreting gestures or body language. That way language barrier will not be in the way of sharing ideas and combining brain power.

Materials – Instructional and Technological Needs (attach worksheets used)

Worksheet Pizza, Pizza, Pizza and computer lab for students to graphically represent their data.

Teaching & Instructional Activities

Time	Teacher Activity	Student Activity	Purpose
5 mins.	Hand out worksheet and instructor students to read the worksheet, while the teacher asks the students what comes to mind when I say think pair share and explain how the formative assessment process is going to work.	Students will read the worksheet and be ready to discuss expectations of solving the problem on the worksheet.	Clarify the learning target.
10 mins.	Break into groups of 4. Teacher will remind the students of how to identify proportional relationships and find the unit rate. Instruct students to share their prediction solution and type of function that will represent the pizza problem.	Students will discuss problem facts, needs, and assumptions in their groups and then write predictions on the worksheets individually. One person from each group will report to the whole class	Students will work in groups to use vocabulary to understand the problem and plan for solving the problem.

		predictions and any clarifications.	
10 mins.	Teacher will walk around making sure students are performing the correct mathematical procedures to find a unit rate. Teacher will call on a student from a group to report known facts, needed facts, and assumptions about the pizza problem. Then the teacher will ask a student from another group how this information can be used to create an equation, table and graph. During this discussion teacher will use correct syntax and vocabulary to deep student learning.	Students will create their pizzas. They will identify if it is indeed a proportional relationship individually and proceed by brainstorming a possible unit rate to write the cost of each slice of pizza on their model. Then they will pair up and share their ideas and find the exact unit rate. Using the think pair share method the students will then create an equation representing the total cost of the pizza.	Students will monitor their progress by receiving input from peers in their group and be able to check their unit rate and equation by plugging in the total cost of pizza. Teacher will use the student responses to give feedback on correct language use and knowledge of the area of circle sector
10 min.	Teacher will walk around making sure technology does not stand in the way of learning and formatively, verbally assess students by asking them what a coordinate on the graph represents.	Using their equation and pizza model students will create a table and graph on Geogebra. The graph must have accurate labels and visible points.	Students will use their knowledge of graphing and proportional relationships to solve the problem.
10 mins.	The teacher will discuss the data and then instruct the students to write a solution to the pizza problem and justify their solution with an explanation of how this problem is related to the unit rate and proportional ratios. The teacher will move through the room to support struggling students in completing the worksheet.	Students will be ready to explain to the class how they came up with their equation and how the unit rate and proportional relationship is accurately modeled on their graph. They must also identify a point on the proportional relationship graph and explain what it means in terms of the pizza problem.	Students will write their solution and justify their solution by generalizing it to a more general equation and then support it with a visual representation.

Pizza Worksheet

Construct a pizza with 8 evenly proportioned slices.

8 slices is worth \$14.80

What is the unit rate? (cost of one slice)

Make a table in Geogebra representing cost of slices 1,2,4,6,8

Based on the unit rate write an equation representing the total cost of the pizza. Let y be total cost and x be the number of slices.

Now graph the equation on Geogebra make sure your points line up with your table values.

Attach your graph to the back of this worksheet

Benchmark Assessment to be given the following day to make sure students remember the information taught and are able to apply the mathematical procedures to another scenario.

Define Ratio, proportional relationship, and unit rate.

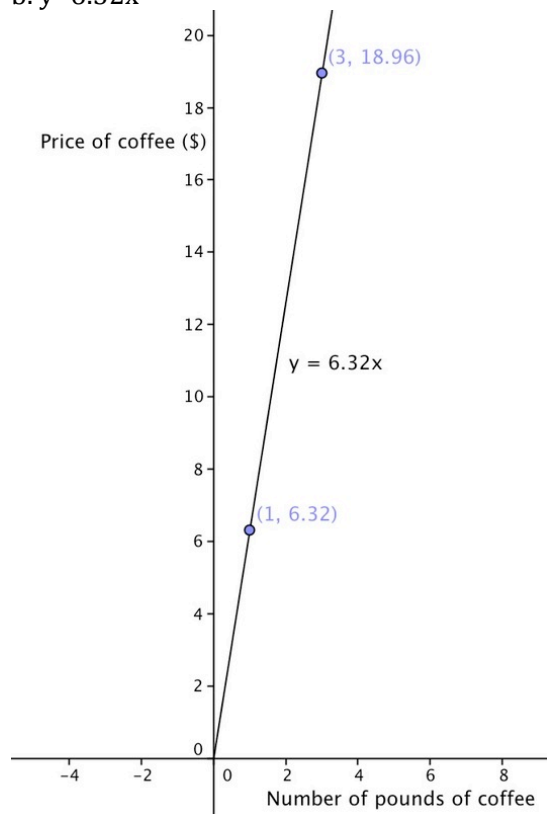
Thinking about these definitions and use them to your advantage in answering the following questions.

Coffee costs \$18.96 for 3 pounds.

- What is the cost for one pound of coffee?
- At this store, the price for a pound of coffee is the same no matter how many pounds you buy. Let x be the number of pounds of coffee and y be the total cost of x pounds. Write an equation to represent the scenario.
- Draw a graph of the relationship between the number of pounds of coffee and the total cost.
- Where can you see the cost per pound of coffee in the graph? What is it?

a. You can find the cost for one pound of coffee by dividing the total cost by 3. Coffee costs \$6.32 per pound.

b. $y = 6.32x$



c. We may graph the proportional relationship between the total cost and the number of pounds by plotting the line through the origin and (3, 18.96). (graph below)

d. The cost of one pound, \$6.32, may be seen on the graph in two ways:

- As the point $(1, 6.32)$

As the slope of the line: \$6.32 per pound.