Excel, Geometry

Overview

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. The CCSS (Common Core State Standards) for the trigonometric aspect of circles in geometry tie in very closely with the CCSS for high school function in trigonometry, for this reason we are going to take a step into trigonometric functions to help students better understand the geometry part of trigonometry. Students begin their study of trigonometry with right triangles.[G-SRT.6] Right triangle trigonometry is concerned with ratios of sides of right triangles, allowing functions of angle measures to be defined in terms of these ratios. This limits the angles considered to those between 0° and 90°. This section briefly outlines some considerations involved in extending the domains of the trigonometric functions within the real numbers. Traditionally, trigonometry includes six functions (sine, cosine, tangent, cotangent, secant, cosecant). Because the second three may be expressed as reciprocals of the first three, this progression discusses only the first three. This unit will focus on the cluster of extending the domain of trigonometric functions using the unit circle. In this unit students will be introduced to radians and why they are important, understanding how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, and the use of special right triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π - x, π + x, and 2π – x in terms of their values for x, where x is any real number.

For this progression we are working with alternative high school students, we are not given a textbook or specific curriculum to follow. Luckily, we have the CCSS which we can follow and make our own curriculum from. The following progression contains three lesson for which students will understand radian measures of an angle as the length of the arc on a unit circle, learn to explain the unit circle in the coordinate plane, use special triangles to determine geometrically the values of sine, cosine, and tangent for$\frac{\pi}{3}, \frac{\pi}{4}, and\ \frac{\pi}{6}$.

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Traditionally, trigonometry concerns "ratios." Note, however, that according to the usage of the Ratio and Proportional Relationships Progression, that these would be called the "value of the ratio." In high school, students' understanding of ratio may now be sophisticated enough to allow the traditional "ratio" to be used for "value of the ratio" in the traditional manner. Likewise, angles are carefully distinguished from their measurements when students are learning about measuring angles in Grades 4 and 5. In high school, students' understanding of angle measure may now allow angles to be referred to by their measures.

Trigonometric Functions

What is a radian?

In trigonometry circular angles are usually measured in radians rather than in degrees. Radian measure is defined so that in the unit circle the measure of an angle is equal to the length of the intercepted arc.[F-TF.1] Students will be given the equation for turning degrees into radians and vice versa. As a class students will be given different examples to perform on changing degrees and radians. An angle of measure 1 radian is approximately 57.3 degrees. A full rotation around a circle is 360 degrees, which turns out to be an equal measure to the circumference of the unit circle, or 2π. In order for the students to make these types of connections they will be given two equations of convergence (degrees to radians (DtR) and radians to degrees (RtD)) DtR: $x \ * \frac{\pi}{180}$ where x is the given degree, and RtD: $x * \frac{180}{\pi}$ where x is the given radian.
In geometry, students learn, by similarity, that the radian measure of an angle can be defined as the quotient of arc length to radius.[G-C.5] As a quotient of two lengths, therefore, radian measure is "dimensionless." That is why the "unit" is often omitted when measuring angles in radians.
There are two key benefits of using radians rather than degrees:

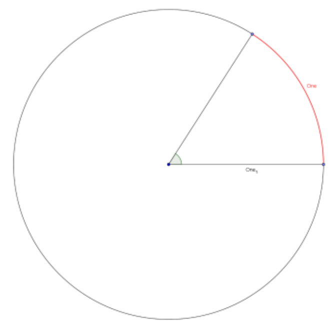
- Arc length is simply $r\theta$, and
- Sin $\theta \approx \theta$ for small θ.

In calculus, the benefits of radian measure become plentiful, leading, for example, to simple formulas for derivatives and integrals of trigonometric functions.

Explain the unit circle:

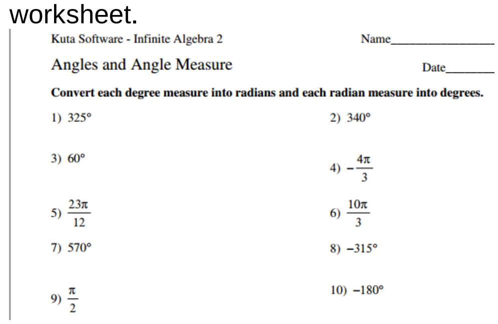
Consistent with conventions for measuring angles, counterclockwise rotation is associated with angles of positive degree measures and clockwise rotation is associated with angles of negative degree measures. In right triangle trigonometry, angles must be between 0 and 90 degrees. Circular trigonometry allows angles that describe any amount of rotation, including rotation greater than 360 degrees.[F-TF.2] With the help of a diagram, students mark the intended angle, θ, measured counterclockwise from the positive ray of the x-axis. During this lesson students will be asked to determine the marked angle, θ, measured counterclockwise from the positive ray of the x-axis; identify the coordinates x and y; draw a reference triangle; and then use right triangle trigonometry. In particular, sinθ = y/1 = y, cosθ = x/1 = x, and tanθ = y/x. Given a radius of 1, student will be able to compute values

F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.



G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Benchmark Assessment: Conversion worksheet.



Kuta Software - Infinite Algebra 2 Name______________

Angles and Angle Measure Date________

Convert each degree measure into radians and each radian measure into degrees.

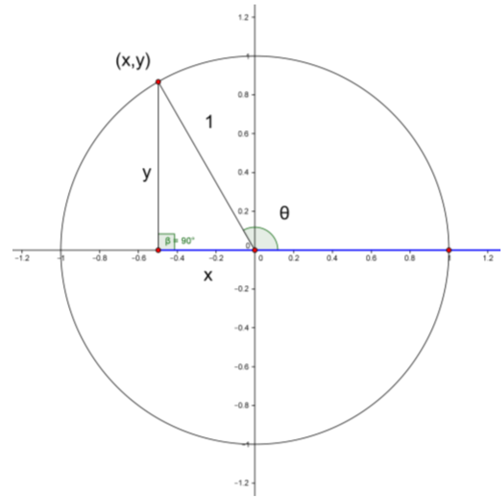
1) 325°
2) 340°
3) 60°
4) $-\frac{4\pi}{3}$
5) $\frac{23\pi}{12}$
6) $\frac{10\pi}{3}$
7) 570°
8) −315°
9) $\frac{\pi}{2}$
10) −180°

F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

of any of the trigonometric functions. This way, students can compute values of any of the trigonometric functions, being careful to note the signs of x and y. In the figure as drawn in the second quadrant, for example, x is negative and y is positive, which implies that sin θ is positive and cos θ and tan θ are both negative. Students will be given radian measures and asked to draw them out onto unit circles and create a right triangle to determine sinθ, cosθ, and tanθ.

Special Right Triangles and the Unit Circle:

The final lesson is sometimes known as "unwrapping the unit circle." On a new set of axes, the angle θ is plotted along the horizontal axis and one trigonometric functions is plotted along the vertical axis. With the help of the special right triangles, 30°-60°-90° and 45°-45°-90°, for which the quotients of sides can be computed using the Pythagorean Theorem, [8.G.7] the values of the trigonometric functions are easily computed for the angles $\frac{\pi}{3}, \frac{\pi}{4}, and\, \frac{\pi}{6}$ as well as their multiples.[F-TF.3(+)] Students need to develop fluency with the trigonometric functions of these special angles to support the "unwrapping of the unit circle" to create and graph the trigonometric functions.



Note that this convention for measurement is consistent with conventions for measuring angles with protractors that students learned in Grade 4. The protractor is placed so that the initial side of the angle lies on the 0° - mark. For the angles of positive measure (such as the angles considered in Grade 4), the terminal side of the angle is located by a clockwise rotation. See the Geometric Measurement Progression.

Benchmark Assessment: In their own words students will explain the use of the unit circle and why they think it is important to learn about.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

F-TF.3(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π - x, π + x, and 2π – x in terms of their values for x, where x is any real number.

Lesson Title: Degrees and Radians

Unit Title: Unit Circle

Teacher Candidate:

Subject, Grade Level, and Date: Geometry, Juniors and Seniors,

Placement of Lesson in Sequence

The students have previously been learning about Sine, Cosine, and Tangent, and how to find degrees of right triangles using sin, cos, and tan. Now students will be learning about the importance of radians and how they are connected to degrees of right triangles in, what we call, a unit circle.

Central Focus and Essential Questions

This lesson introduces concepts that are the prerequisites to defining trig functions on the unit circle. Radian measure and converting between degrees and radians are introduced. Students will also learn how to draw and measure positive and negative angles in standard position, identify and draw co-terminal angles, and determine a reference angle.

Content Standards

CCSS.Math.F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Learning Outcomes	Assessment
• Students will be able to draw and identify angles in standard position in both degree and radian measure. • Students will be able to take a given degree or radian and convert it into a radian or degree. • Students will understand radians in terms on arcs on a given circle (the unit circle)	Students will be given a benchmark assessment (a worksheet) asking them to convert a given arc angle measured in degrees and convert it to radians.

Learning Targets	Student Voice
• I know what a radian measure is in terms of a unit circle • I know how to convert from degrees to radians and vis-versa	At the beginning of the class period students will be told that the days learning target is to learn about radians. Students will be asked to volunteer to work on different examples of conversion to show they're understanding of radians. Students

• I know how to draw an arc from standard position given a radian or degree measure.	will be given a worksheet to assess their individual achievement towards the learning target.

Prior Content Knowledge and Pre-Assessment

Students are able to solve a right triangle using the Pythagorean Theorem.
Students are able to solve special right triangles.
Students are able to multiply and divide with numbers containing radicals.
Students are able to order sides of a triangle from smallest to largest when knowing the angles of a triangle.
Students are able to determine the largest or smallest ratio of sides of a triangle when the angles are known.

Academic Language Demands		
Vocabulary & Symbols	**Language Functions**	**Precision, Syntax & Discourse**
• Radians (π) • Degrees (°) • Unit Circle • Sine, Cosine, Tangent • Arc measure	• Students will identify terms using proper language. • Students can explain the difference between two terms. • Students will use their understanding of a term to complete examples given by the teacher and worksheet in class.	**Mathematical Precision:** Students must be able to convert radians to degrees and degrees to radians. Students must be able to draw a given arc measure in degrees or radians. **Syntax:** Students must be able to use a protractor to measure arc angles. Students must use the given equations to solve for radians or degrees (To convert from degrees to radians, multiply by $\frac{\pi}{180°}$. To convert from radians to degrees, multiply by $\frac{180°}{\pi}$) **Discourse:** Students must be able to verbally explain their step by step process to convert angles, and how they drew given angles.

Language Target	Language Support	Assessment of Language Target
- I can convert degrees to radians and vice versa. - I can draw an arc from standard position given a specific degree or radian.	Throughout the lesson students and the teacher will be using their mathematical vocabulary. Students will be given a worksheet assessing their understanding of radians.	Students will be given verbal feedback when the volunteer to answer questions on the white board. Students will turn in their worksheet as a benchmark assessment for the unit. Worksheets will be graded and given back to students for them to receive feedback about common misconceptions, and see their own level of knowledge. At the end of the unit students will be quizzed on their knowledge of the overall unit.

Lesson Rationale (Connection to previous instruction and Objective Standards)

Differentiation, Cultural Responsiveness and/or Accommodation for Individual Differences

All the students are part of an alternative high school that is able to take more time to help students understand each lesson and help them gain the knowledge necessary to move onto better things.

Materials – Instructional and Technological Needs (attach worksheets used)

Whiteboard markers to answer questions

Protractor

Pencil

Straight edge

Radian Worksheet

Teaching & Instructional Activities			
Time	**Teacher Activity**	**Student Activity**	**Purpose**

9:00	Teacher will introduce the day's lesson and activity. Explain what a radian is and the importance of radians compared with right triangles and the unit circle.	Takes notes of definitions and equations, ask questions to help understanding.	To get students focused on the day's content area and thinking about radians and what they think they'll need to know.
9:10	Teacher will write down a few examples for students to volunteer and answer about convergence of radians to degrees and vice versa.	Students will volunteer to answer questions given by the teacher and verbally explain the steps to the process they used to answer the equation.	Students show the knowledge they just received and see for themselves how the convergence equations work
9:25	Teacher hands out the worksheet for students to begin working on.	Students begin working on their worksheet to show their knowledge and understanding of radians.	Benchmark assessment of student knowledge.