## Veronica Guadarrama <br> Math 499E <br> Learning Progression Math CCSS Cluster

February 25, 2015

## $7^{\text {th }}$ grade, Statistics and Probability

## Overview

Several domains in $6^{\text {th }}$ grade are important in preparing students for probability and statistics in the $7^{\text {th }}$ grade and they are: recognizing and understanding a statistical question, understanding how to collect data to report observations, and describe data distribution using various types of statistical representations (i.e. plots on a number line, histograms, and box plots). In this progression, students learn how to investigate change process and develop, use, and evaluate probability models.

The statistics and probability standards are luxuriant for the standards for mathematical practice. In particular, three mathematical practices that stand out are MP1: Make sense of problems and persevere in solving them, MP2: Reason abstractly and quantitatively, and MP4: Model
 with mathematics. Students are expected to analyze given relationship and make conjectures about the form and the meaning of the solution as well as quantitatively reasoning the given representations and creating models to fit the context situation. A reference and resource you can use is the Gelencoe Mathematics Pre-Alegebra book Chapters 6 and 12.

Throughout this learning progression students will be taught using a best practice called differentiated curriculum. Differentiated curriculm is a practice "where teachers adapt the curriculum in different ways to meet the needs of all their students. The content taught, the process used, the product expected, or the physical factors of the environment created may be modified to help students achieve success. Task choices and flexible grouping may be used to accommodate background knowledge and interests of small groups or individual students." (http://www.ncpublicschools.org/docs/curriculum/bpractices2.pdf). This type of instruction will benefit my students because it will give them the opportunity to to have each of their individual needs met as well the opportounity to have their learning modified if they are struggling understanding a certain concept or topic. For the students in my classroom that struggle to learn through a very structured and direct method of teaching, learning the concept through direct instruction and then having an opportunity to practice the concept through the activities with their peers in various types of researchbased grouping such as Pairs Check, Jigsaw, Think-Pair-Share, Team Product, https://www.srri.umass.edu/topics/collaborative-group-techniques and then though the benchmark assessment question (also can be referred to as hinge problem), students show they have met the standard and are ready to move on.

This best-practice is great for a classrooom where students work the best in groups and where students' backround knowledge and interests are the driving factor in the types of activies and tasks that are given to students. The activities and benchmark assessments were created for a classroom of students who enjoy learning through a hands-on approach and with activites. The activities and benchmark assessments were created so that they would meet a varied range of student interests and prior knowledge, but the activies can also be restructured if they do not fit your students.

Define and understand probability Students have been working with statistics and probability all their lives in games. For example, when playing a game that uses a spinner such as Twister, students have seen equal outcomes. Such as the probability of getting either a foot move is $1 / 2$ of the spinner and the probability of getting a hand question is $1 / 2$ of the spinner also. If a student was looking at the probability of getting left hand spun the probability would be $1 / 4$ as would be the probability of getting right hand, left foot, or right foot. If a student wanted to know what would be probability of getting the outcome right hand yellow they would have to count the probability of getting right hand yellow $=1$ and then divide it by total outcomes $=1 / 16$. However, even if students may have pondered probability questions before, they have never formally been taught about probability. In this lesson students will learn the definition of probability and be able to distinguish and solve different probability questions through a hands-on Rock, Paper, Scissors activity and through the homework benchmark assessment.

Nevertheless, before this lesson students need to have some prior knowledge. My students' prior knowledge consists of understanding the definition of fractions, equivalent fractions, and can add, subtract, multiply and divide fractions. This prior knowledge can serve as a foundation for learning because it has help students understand the definition can concept of probability by relating it to prior
 knowledge of fractions. By the end of the lesson students should understand that the probability of an event is always between 0 and 1 , inclusively. Meaning that 1 is the largest probability that you can have if you're just talking about one whole. By the end of the lesson students will also be able to explain that the closer the probability is to 1 , the more likely it is to occur.

## Learning the Concept:

In this first lesson teachers will teach the definition of probability explaining that probability is ratio that describes the total outcomes event and the chance that the outcome desired will occur When teaching this lesson a teacher can use the Pre-Algebra book by McGraw-Hill Education, Chapter 6 on page 262 to introduce the definition of probability (as directly expressed below) and students can easily follow along with examples in the book, which would be a big help for students that are visual learners or have special learning needs. When teaching students about probability, one of the best ways is to start is with a probability scale. The probability scale shown below allows students to visually gain a basic understanding of the probability range and serve as guide for this introduction before they go deeper into this learning progression into deeper concepts about statistics probability. Afterwards, through asking questions such as "what is the probability of gravity failing?" using the probability scale correctly, the teacher understands that the students understand this portion of the lesson and are ready to move on. If students are ready to move on, then, a teacher should seaway into explaining to students how to complete a tree diagram from different scenarios which can be found in the Algebra book, or which can be created by the teacher to best fit individual student's needs. Teaching students to create tree diagrams is a great way to show them how they can... Another great way to teach students... By the end of the lesson in order to meet the standard, students should be able to explain that the probability of an event is always between 0 and 1 only and that the closer the probability is to 1 , the more likely that the event or outcome is to occur.

## Book Definition:

Probability: The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.
$P($ event $)=$ number of favorable events/number of possible events

|  |  | Probability |  |  |
| :---: | :---: | :---: | :---: | :---: |
| less ofter tharinot |  | $\longrightarrow$ | more often $\qquad$ |  |
| $\square$ | 1 |  | 1 |  |
| 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
| o.o | 0.25 | 0.5 | 0.75 | 1.0 |
| 0\% | 25\% | 50\% | 75\% | 100\% |
| impassible | unlikely | equally likely | likely | certain |

## 7.SP.C.5. Activity:

After teaching students the definition of probability, a great way to allow students to begin to further deepen their conceptual understanding of probability is through a fun and engaging activity. This Rock, Paper, Scissors-The Study of Chance Game activity found on pg. 7 helps students to learn if a game that they may have much experience with, to learn if it truly is fair. Because, maybe some students believe that it might not be exactly fair. By playing this game students are proving that in general the probability of getting each outcome: rock, paper, or scissors is in fact equal at $1 / 3$ probability for each. After this activity, students will be able to prove that the game is truly fair and be able to interpret and display the data obtained.
This activity is inclusive, because the game rock, paper, scissors is very popular and a majority of students are aware of this game and know how to play it and, for those who do not know how to play, it a simple game to learn. A teacher can support for varied learning by carefully structuring his or her pair groups. By pairing students by heterogeneous ability level students can help each other out during the activity. This type of grouping benefits both students because it allows the lower level student to reach the expected level of learning and the higher achieving student by teaching can deeper their understanding of the conceptual and procedural of probability, because according to a well regarded philosophy, "by teaching we learn". Students will monitor their learning by finishing their game trials, filling out the worksheet, and getting feedback from the teacher throughout the lesson and during the classroom discussion and review of activity questions. Then, students will be given the assessment question shown on the right as a homework assessment that will be due on the next day. This benchmark assessment allows me to assess if students fully understand the basics of probability, for example, if students rate the probability of seeing a dinosaur a 1 , which we know that because dinosaurs are extinct that the probability of seeing a dinosaur is actually closer to zero, then I know that this material needs to be retaught.
*Note the connection with MP3: Construct viable arguments and critique the reasoning of others. Including justify their conclusions, communicate them to others, and respond to the arguments of others during the class discussion of the activity questions. Then, students reason inductively about their rock, paper, scissors, data and make plausible arguments take into account the context of their rock, paper, scissors data.
7.SP.C.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1 / 2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

The assessment below assesses students' ability to express their knowledge of where the probability lies on the probability scale through the asking of questions that are relatable to real-life. Where students use their reasoning and problem solving skills to correctly state the right probability. By being able to pass this benchmark assessment with at least $80 \%$ accuracy students are showing their understanding of the definition of probability and their ability to read a question pertaining to probability and correctly draw or create the given probability; showing their conceptual understanding of the objective: understand that the probability of an event is always between 0 and 1 and be able to use the probability range to express their conceptual understanding of probability.

## 7.SP.C.5. Assessment:

1. Decide where each event would be located on the probability scale.
A. You will see a live dinosaur on the way home from school today. Solution: 0
B. A solid rock is dropped in the water will sink.

Solution 1
C. A round disk with one side red and the other side yellow will land yellow side up when flipped.
Solution: $1 / 2$
D. A spinner with four equal parts numbered 1-4 will land on the 4 on the next spin.
Solution: $1 / 4$
E. Your name will be drawn when a name is selected randomly from a bag containing the names of all of the students in your class. Solutions will vary
F. A red cube will be drawn when a cube is selected from a bag that has five blue cubes and five red cubes.
Solution: $1 / 2$ (equally likely)
G. The temperature outside tomorrow will be
-250 degrees. Solution: 0
2. Design a spinner so that the probability of green is 0 .
3. Design a spinner so that the probability of green is 1 .
4. Design a spinner with two outcomes in which it is equally likely to land on the red and green parts.
5. What do you think it means for an event to have a probability of $1 / 2$ ? Solution: $50 \%$ probability
6. What do you think it means for an event to have a probability of $1 / 4$ ? Solution: $25 \%$ probability

Collecting data, then observing and predicting relative frequency Students begin learning this standard by relating probability to the long-run (for more than five or ten trials) to relative frequency of a chance event using items such as coins, number cubes, cards, and spinners. In order to teach a varied number of students, handson activities when students collect the data for probability experiments are crucially important. Once students have developed a clear connection between the observed relative frequency and theoretical probability, if a teacher has the technology, they can then move to simulating probability experiments using technology such as graphing calculators or computers. Students develop a clear connection between the observed relative frequency and theoretical probability by understanding that the observed relative frequency comes from an actual experiment that is done and the recorded favored outcomes that are obtained is called the observed relative frequency; whereas the theoretical probability is the probability that is expected to occur based on theory and without having conducted a scenario. For example, when teaching students this concept a good way to start might be by discussing the probability of heads or tails. As a class, for example, you can explain that the theoretical probability of getting heads when you have a quarter is said to be $1 / 2$ and the theoretical probability of getting tails is also $1 / 2$ because the coin is fair. But, then practice you could have the class split into pairs of two and record their outcome values for flipping the coin at least 50 times. At the beginning students should obtain values where one side is "winning" or is occurring more frequently, but towards the end of their trials their observed frequencies for each side should be approximately $1 / 2$ or fair. Proving that yes, the flipping of a fair coin is truly equal and the observed relative frequency in this case matches the theoretical probability, but sometimes, this not always the case.
In order for students to understand this standard they must understand the connection between relative frequency and theoretical probability. Using the curriculum book examples on page 310 and 311 as a direct teaching guide, you can efficiently show students that: if you know the structure of the generating mechanism (e.g., a bag with known numbers of blue and green marbles), you can anticipate the relative frequencies of a series of random selections from the bag with replacement. After understanding that in the bag there is only is only a set number of outcomes and that these outcomes are called the sample space and understanding that the sample space defines the whole outcomes (previously called possible outcomes in previous lesson). However, if you do not know the structure (e.g., the bag has unknown numbers of blue and green marbles), you can approximate it by making a series of random selections and recording the relative frequencies.

## 7.SP.C. 6 Activity:

After students have had the opportunity to practice the standard through the flipping of the quarter, students can further practice seeing how long term probability turns into relative frequency of a chance event by rolling a dice 50 times and recording its values for each number. Prior to doing this experiment though, students will conceptualize based on their sample space, how often they believe each number should occur. Having the opportunity to brainstorm their ideas on the outcomes, students are practicing their ability to explain the theoretical probability to their experiment and enhancing their ability to reason complex mathematical scenarios. Then, by doing this activity students are learning how to collect data in a probability experiment, which is crucially important in daily life and potentially a future career.
During the activity each pair of students has dice and will roll it 50 times and record the values that come up on a table where the frequency of each number is recorded using tally marks. After all the data is collected students will explain the long-run relative frequency of each number rolled. Then, students will use this table results to predict the approximate relative frequency that a 3 or a 6 would come up if they were to roll the cube 600 times. This activity also helps students to practice their procedural understanding on the concept by showing their ability to correctly state the relative frequency of the numbers 3 and 6 . This activity is found on page 8 .
*Note the connection with MP 2: Reason abstractly and quantitatively. Including in the activity students make sense of quantities and their relationships in the dime problem, because they show their ability to decontextualize the given situation and represent it symbolically on their table and then contextualize the solutino to find the long-run relative frequency of flipping a dime 600 times.
7.SP.C.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Examples of student strategies for generalizing from the relative frequency in the simplest case (one sample) to the relative frequency in the whole population are given in curriculum text on page. 310.

After students have learned the concept, practiced the concept through various different activities through the lesson, for an exit slip students will complete the following benchmark assessment. This assessment question serves a hinge question and allows the teacher to make correct decisions on his or her students' understanding of the concept and to decide if the students are ready to move on, need to review the concept more but just briefly, or if the students have several misconceptions, misunderstandings and need to be retaught.

This assessment question is a great hinge problem, because students collect data by flipping two coins (or theoretically flipping two coins) in order to observe or predict what are the outcomes (sample space) obtained by the of rolling these two coins. Then, after students have found the sample space, they are asked what is the relative frequency of obtaining one of the solutions and students should be able to express the probability of that occurrence. After correctly solving the hinge problem students show they are ready to continue to the next concept.

## 7.SP.C.6. Assessment:

Show using more than one way the possible outcomes of flipping two coins. Then, explain what is the relative frequency of getting two heads.

Potential Solutions:
*Students can choose to show thier answer as an organized list (as the response in the far left), as a table (as the response in
the middle), or as a tree diagram (as response on the far right)


All the possible outcomes of the toss of two coins can be represented as an organized list, table, or tree diagram. The sample space becomes a probability model when a probability for each simple event is specified.

Solution Cont:
Relative probabaility of rolling two heads from two coins is $1 / 4$.

Probability modeling When students are learning about probability modeling, they are really finding a probability for each possible non-overlapping outcome for a chance process so that the total probability over all outcome is unity, which is also called the sample space for the model. The sample space becomes a probability model when a probability for each simple event is specified. For example, the sample space for the toss of two coins (fair or not) is often written as $\{\mathrm{TT}, \mathrm{HT}, \mathrm{TH}, \mathrm{HH}\}$ which produces the following trial results: HH, HT, TH, TT, HT, HH, HH, TT, TH, and TT. These possible outcomes of two coins toss can be represented as an organized list, table, or tree diagram. Different representations can be either theoretical (based on the structure of the process and its outcomes) or empirical (based on observed data generated by the process).
${ }^{7 . S P . C .7 .}$ Activity:
Students determine what kind of experiment they need to conduct, then conduct the experiment and report out on the results they find. Students will work at different centers established in the room. Each group will receive one of the "What's the Chance" cards (found on page 9). Each card describes a situation and students will design an experiment and choose from the tools that are available to conduct the experiment. They will be able to choose from different spinners, number cubes, polyhedral dice, color tiles, centimeter cubes, coins, etc. If the chance card is one asking for simple probability, students may only need a spinner to conduct the experiment. Depending on the number of options in the chance card, students may need a polyhedral die and a number cube. Part of the design and discussion will be for the groups to decide what is needed. After determining how to conduct the experiment, groups will find the theoretical probability for their experiment. They need to create a display for the possible outcomes. When the experiments are finished, student groups will report out to the class about their card and the experiment they conducted. This activity is great because it promotes deeper thinking and creates a "challenging environment" for students, but still gives struggling students enough manipulatives for them to easily grasp the concept.
*Note the connection with MP 2 Reason abstractly and quantitatively. Including abilities to bear on problems involving quantitative relationship and the ability to decontextualize and contextualize.
7.SP.C. 7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Examples of student strategies include using the assessment question as an end of teacher instruction question, working with their peers during the activity, and teacher feedback during the activity.

## 7.SP.C.7 Assessment:

In the toss of two balanced coins, what can you say about the four outcomes of the sample space? Please answer in at least two complete sentences.


## Solution:

Since the coin is balance, each outcome is given the equal theoretical probabilities of $1 / 4$ because of the symmetry of the process; meaning that the outcome of heads is just as likely as an outcome of tails.

Using uniform probabaility models to find probability To meet this standard, students must understand that if you have uniform probability model (each item in the situation is equally likely), then, the outcomes are equally likely. For example, as a warm up problems the teacher can ask students the following two questions: What is the probability of randomly selecting a name from a list of ten students? (Solution: 0.10 or $10 \%$ ) And, if there are exactly four seventh graders on the list, the chance of selecting a seventh grader's name out of the class is what? (Solution: 0.40 or $40 \%$ ). As a reference, teachers can refer to Glencoe Mathematics Pre-Algebra book Chapter 12-9 on pages 650-655. In this lesson, students work with a uniform probability model in the "How Many Buttons? Activity" and use it to find probabilities of events and then they compare their observed probability frequencies to answer questions pertaining to randomly selection. Where students will reason if the agreement is good or not good; if it is not good, they will explain possible sources of the discrepancy. Which, through this discussion my students are demonstrating their conceptual understanding of the common core standard and also showing their use of their reasoning skills; allowing me to more easily assess students' progress towards the achievement of the standard.
${ }^{7 . \text { SP.7.a. How Many Buttons? Activity }}$
Students will look at the shirt you are wearing today, and determine how many buttons it has. Then, students complete a table for all the members in the class using tick marks to keep a running total in each box. Thereafter, each student will write his or name on an index card and one card will be selected randomly By answering several "randomly selecting" probability questions. This activity can be found on page 10 of this learning progression. The purpose for doing this activity is to have students use a uniform probability model which is given to them as a method of practicing their ability to see that even though each person's shirt may have different buttons, the model itself is uniform because the probability that a shirt will have one, two, three, or more buttons, is exactly the same. Therefore, this scenario even though it was random selection it is a probability model that is called uniform.

Observing frequencies to find non-uniform probability Sometimes probability is not uniform, meaning that the probability of each outcome is not equally likely. For example, the that probability a tossed thumbtack landing 'point up' is not necessarily $1 / 2$ just because there are two possible outcomes; these outcomes may not be equally likely and an empirical answer can only be found by tossing the tack and collecting data. In this lesson, students can be put into pairs by ability level, so the 'higher-achieving student' can assist the 'lower-achieving student'. And together, work toward completing the "Spinning Penny and Cup Activity." This type of grouping benefits both students because it allows the lower level student to reach the expected level of learning and the higher achieving student by teaching can deeper their understanding of the conceptual and procedural of probability, because according to a well regarded philosophy, "by teaching we learn."
${ }^{\text {7.SP.7.b. }}$ Spinning Penny \& Cup Activity:
In this activity, students will find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down by observing the results of the data on the table below and answer the following question: Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? Through this activity students are learning gaining an understanding of the fact that the probability obtained from a tossed paper cup will not always necessary be equal, so therefore, the probability that a penny will land heads or that a tossed paper cup will land openend down is called an uniform probability. It is important for students to understand the difference between uniform probability and non-uniform probability, because unlike in uniform probability, in uniform probability the outcomes will not always be necessarily equally likely.

| Trial \# | Outcome of penny | Outcome of cup |
| :--- | :--- | :--- |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 20 |  |  |

*Note the connection with MP 1: Make sense of problems and persevere in solving them. Including explaining the problem to themselves, understanding the meaning of a problem, and looking for entry points to its solution. Then, in the button activity students analyze the given scenario, constraints, relationships, and goal in order to solve "randomly selecting" probability questions.
${ }^{7.5 P}$.7.a. $D e v e l o p$ a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

## 7.SP.7.a Assessment

Out of a classroom that has eight students: Omar, John, Drew, Jason, Wyatt, Jane and Kelly, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
Solution:
Probability $($ Jane being chosen $)=1 / 8$
Probability $($ girl being chosen $)=2 / 8=1 / 4$
Probability (Jane will be selected \& girl will be selected $)=1 / 8$

* $1 / 4=1 / 32$
*Note the connection with MP 1: Make sense of problems and persevere in solving them. Including explaining the problem to themselves, understanding the meaning of a problem, and looking for entry points to its solution. Then, in the penny and cup activity students analyze the given scenario, constraints, relationships, and goal. In order to find through observations what are the outcomes of a spinning penny will land heads up or that a tossed paper cup will land open-end down out of the sample space.
7.SP.C.7.b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.


## 7.SP.7.b Assessment

Out of a classroom that has eight students: Omar, John, Drew, Jason, Wyatt, Jane and Kelly, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

## Solution:

Probability (Jane being chosen) $=1 / 8$
Probability $($ girl being chosen $)=2 / 8=1 / 4$
Probability $($ Jane will be selected \& girl will be selected $)=$
$1 / 8 * 1 / 4=1 / 32$

Probability of compound events In this lesson students learn about the probability of compounded events. By the end of this lesson students should understand that the probability of compounded events is found by: counting the outcomes for chance event (both uniform and non-uniform probability) in each individual probability and then multiplying the chance events together in finite situations. Students also should gain experience using diagrams, especially trees diagrams and tables, for organizing and counting possible outcomes from chance processes. For example, the 36 equally likely outcomes from the toss of a pair of number cubes are most easily represented by a two-way table. After the basics of probability are understood, students can experience setting up a model and using simulation to collect data and estimate probabilities for real situations that are somewhat complex.

## 7.SP.8. Red, Green, or Blue? Game:

This is a game for two people (found on pages 1-13). Each student cuts out three dice (found on page 14); one is red, one is green, and one is blue. These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other. The numbers on each of the dice are shown below.


After the dice are cut out, to play the game, each student picks one of the three dice. However, they have to pick different colors. Then, both students roll their dice. The highest number wins the round and then the results are recorded on their individual data tables. The players roll their dice 30 times, keeping track of who wins each round. Whoever has won the greatest number of rounds after 30 rolls wins the game. Then students answer the following questions pertaining to the probability of compound events that are uniform: Who is more likely to win when a person with the red die plays against a person with the green die? What about green vs. blue? What about blue vs. red? Would you rather be the first person to pick a die or the second person? Explain. (Solution is found on page 13 of this learning progression).
7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
7.sp.8. Assesment

If a bag contains $2 \overline{\text { blue marbles and }} 3$ red marbles, then the probability of selecting a blue marble, replacing it, and then selecting a red marble is ?

## Solution:

Probability of selecting blue marble $=\mathrm{P}(\mathrm{A})$
Probability of selecting blue marble $=\mathrm{P}(\mathrm{A})$
$\mathrm{P}(\mathrm{A}) * \mathrm{P}(\mathrm{B})=2 / 5 * 3 / 5=6 / 25$
7.SP.C.8.a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

## 7.SP.8.a. Assessment:

If you flip three fair coins, what is the probability that you'll get a head on the first flip, a tail on the second flip, and another head on the third flip? Solution:
The tree diagram below shows all of the possible outcomes of flipping three coins.


Sample spaces for compound events In prior lessons students, students have learned the definition of sample space and have learned how find the probability of individual events. Now, students will learn how to find probability from compound events (which is one or more different events-can be uniform or ununiformed). For examples and practice problems please reference the Glencoe Mathematics PreAlgebra book Chapter 12-9 on pages 650-658.

## 7.SP.8.b. Activity:

You're playing a mixed up version of Candy Land where all the characters and their treats are out of order. In this game you get to choose your character and the treat you want to use to play the game shown on a table. In this activity students answer the question of: If you randomly choose a character and their candy, what is the probability that you will get the correct character and their candy out of the total?

Design \& use a simulation to generate frequencies for compound events After students have learned about compound events student, students will work with simulation problems to find the frequencies for compound events. For example, students can answer the following question: Suppose, over many years of records, a river generates a spring flood about $40 \%$ of the time. Based on these records, what is the chance that it will flood for at least three years in a row sometime during the next five years? In this lesson to support varied learners, a teacher can put students into pairs based on prior formative assessment data where the teacher pairs students that have shown more evidence of understanding the learning progression with a student who is struggling more. If students are kinesthetic learners, the teacher can give students representations of the scenarios such as a deck of cards so students can simulate their own outcomes.
7.SP.8.c. Activity:

A medicine has 3 in 5 chance of curing the condition for which it is prescribed. If two patients are chosen at random, the probability that the medicine will cure both of them is desired. A simulation is set up to determine the experimental probability of this occurring.

The table shows results of using five cards. The 1 (ace), 2 and 3 of hearts represent the medicine curing the condition and the 4 and 5 of hearts represents the medicine not curing the condition.

| Trial Number | Cards Drawn |
| :--- | :--- |
| 1 | 3,4 |
| 2 | 4,1 |
| 3 | 5,3 |
| 4 | 5,5 |
| 5 | 1,2 |
| 6 | 2,4 |
| 7 | 1,4 |
| 8 | 3,2 |
| 9 | 4,5 |
| 10 | 4,3 |
| 11 | 4,5 |
| 12 | 3,2 |
| 13 | 4,4 |
| 14 | 2,4 |
| 15 | 4,2 |
| 16 | 4,4 |
| 17 | 4,5 |
| 18 | 2,5 |

What is the number of trials in which the medicine cured both patients?
a) 7
b) 3
c) 9
d) 16
e) 1
${ }^{\text {7.SP.8.b }}$ Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space, which compose the event.

## 7.SP.8.b Assessment

You draw a card from a standard deck of playing cards. What is the probability that the first card you pick will be a black nine or any heart?


Solution:
Probability of getting a black nine $=2 / 52$
Probability of a heart $=13 / 52$
Probability that the first card you pick will be a black nine or any heart $=15 / 52$

[^0]7.SP.8.c. Assessment

1. It is observed that 3 out of every 4 drills make an oil-drilling site strike oil. The probability of striking oil at 7 out of every 10 times must be determined. A simulation can be done to determine the experimental probability.

The table shows the results of carrying out the simulation using a deck of cards. Spades (5) represent not striking oil and all other suits represent striking oil.

| Trial Number | Suits Drawn |
| :--- | :--- |
| 1 | DSHCDHDSDS |
| 2 | DCDCSHDHDC |
| 3 | CHDCDDDHSS |
| 4 | HDHHCHSDSC |
| 5 | CDCCHSDDDS |
| 6 | DSDSDCDSHH |
| 7 | HHSCHDCSSD |
| 8 | DSDSDHCSDD |
| 9 | SDSHSSCSSS |
| 10 | SSCSHSHHHH |

What is the probability of striking oil at least 7 out of 10 times based on this simulation? Answer accurate to two decimal places.
a) 0.64
b) 0.17
c) 0.25
d) 0.80
e) 0.08

## Teacher References

## Activities Relating to Each CCSS in Learning Progression

${ }^{7 . \text { SP.C.5. }}$ Activity:
The purpose of this activity is to introduce basic information on probability and statistics. After this activity, the student will be able to determine whether or not the game is fair and be able to interpret and display the data obtained. The student will also be able to see that probability is used often in society.

Procedures:
Divide the class into pairs and have them play the game fourteen times. A rock is a closed fist. Paper is palm on palm, and scissors is the number two horizontally. The student hits their other hand twice, and on the third time gives the symbol they wish. A rock beats scissors. Paper beats rocks, and scissors beats paper. Instruct the students to keep a record of wins and losses. Then, have students try to answer the activity questions in their groups. After most students have completed both the game and the activity questions, go over the activity questions as a class and ending the class with a class discussion.

## Student Worksheet

## Rock, Paper, Scissors -The Study of Chance Game

Directions:
First, pick if you're going to be Player A or Player B. You and your partner will play rock, paper, scissors eighteen times and record your which hand sign you chose, which hand sign your partner choose, and who won the round after every turn.

How to Play the Game:
A rock is a closed fist. Paper is palm on palm, and scissors is the number two horizontally. The student hits their other hand twice, and on the third time gives the symbol they wish. A rock beats scissors. Paper beats rocks, and scissors beats paper. If the round results as a tie just write down which hand-sign you and your partner both pick, and then denote the round as a tie.

Table for Recording Data:

| Trial/ <br> Round | Player A sign <br> chosen | Player B sign <br> chosen | Outcome (who <br> won) |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| 13 |  |  |  |
| 14 |  |  |  |

## Activity Questions:

Answer the following questions to determine if the game is fair.

1. Drawing a tree diagram, using what you know about how the game is played and won, plus your trial results to find how many possible outcomes the game has.
Solution: 9 possible outcomes per trial/round
2. Label each possible outcome (results) on the tree diagram as to win for $\mathrm{a}, \mathrm{b}$, or tie.
Solution:

3. Count the number of wins for A. Solution: 3
4. Find the probability A will win in any round. Solution: $3 / 9=1 / 3$
5. Count the number of wins for B. Solution: 3
6. Find the probability B will win in any round. Solution3/9 = $1 / 3$
7. Find the mean, mode, and range of your results. Solution: Each answers will be slightly different.
8. Is the game fair? Do both players have an equal probability of winning any round?
Solution: Yes!

## Activities Relating to Each CCSS in Learning Progression

## ${ }^{7 . S P . C .}$ Activity:

Students in pairs will flip a dice 50 times and record their values on a table to find the long-relative frequency each number occurs during the sample space. Then, students will use their trial results to find the approximate relative frequency of a large sample space.

## Procedures:

Each pair of students has a dice and will flip it 50 times and record the values that come up on a table where the frequency of each number is recorded using tally marks. After all the data is collected students will explain the long-run relative frequency of each number rolled. Then, students will use this table results to predict the approximate relative frequency that a 3 or a 6 would come up if they were to roll the cube 600 times.

## Student Worksheet

## Rolling Dice!

## Directions:

Pick a partner and then wait for a dice to be given to you. Then, taking turns, flip the dice a total of 50 times (each person should roll the dice 25 times each) and record values using tally marks into the table below where the frequency of each number is recorded. After you record your data, answer the questions below.

| \# on die | Number of times \# is rolled after 50 trials (Each event is recorded by tally marks) |
| :---: | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

## Long-Run Relative Frequency:

*Make sure to write your answer using fractions which expresses the probability of getting each number out of your 50 trials.

1. Out of 50 trials how many 1 's did you get?
2. Out of 50 trials how many 2 's did you get?
3. Out of 50 trials how many 3 's did you get?
4. Out of 50 trials how many 4 's did you get?
5. Out of 50 trials how many 5 's did you get?
6. Out of 50 trials how many 6 's did you get?

## Predict the Approximate Frequency:

1. Predict the Approximate frequency that a 3 or a 6 will come up if you were to roll the cube 600 times! Show your work in the text box below.

## Activities Relating to Each CCSS in Learning Progression

## 7.SP.C.7. Activity:

In this activity students in groups will determine what kind of experiment they need to do to solve their "What's the Chance" card, then they conduct the experiment, and report their results by creating a display for possible outcomes on a separate sheet of paper. Each card describes a scenario and students design an experiment and choose from several tools that will be provided in order to conduct the experiment.

## Procedures:

Prior to the activity make sure the following materials: different spinners, number cubes, polyhedral dice, color tiles, centimeter cubes, coins, etc. For the activity each group will receive one of the cut out "What's the Chance" cards. Part of the design of this activity and will be for the groups to decide what is needed to solve their situation. After determining how to conduct the experiment, the groups will find the theoretical probability of their experiment. Then, they create a display of the possible outcomes. Finally, when the experiments are finished, the student groups will explain to the class their card, the experiment they conducted, and their answer.

Student Worksheet

## What's the Chance?!

## Directions:

After you receive your "What's the Chance?" card, take some time to discuss with the members in your group to determine which materials you are going to use and how you are going to conduct your experiment. Then, as a group find the theoretical probability for your experiment. Afterwards, create a display for the possible outcomes such as by using a tree diagram or area model to represent your given experiment. When the experiments are finished, your group will explain to the class your card and the experiment conducted.

## Answering the following questions:

- What did you have to consider?
- What tools did you use to help you find the results?
- Were there other options?
- Did your experimental probability match the theoretical probability?
- Were there any surprises in the data?
- Was the experiment fair?
- Overall, make sure you are prepared to justify your reasoning during the discussion. If you are unsure how to approach your question or if have any questions please do not hesitate to ask.


## Question \#1

Joe wants to make some chicken soup but all of the labels have fallen off the soup cans. He knows that there are three cans of chicken soup, two cans of vegetable soup, and four cans of mushroom soup in the cupboard. How like is it that he will pick chicken soup and then design an experiment that you could do to test your prediction. Compare your actual results to your prediction. Explain the differences between your actual results and your predictions.

## Question \#2

Marti is passing out drinks and snacks to the class. She has a box with three flavors of juice boxes and another box with six kinds of chips. Jenny really wants an orange juice box and corn chips. If there is an equal member of each kind of juice and chips in the boxes when Marti gets to Jeremy, what is the chance that he will get the kind of drink and chips that he wants? Predict how likely it is that he will get the kind of drink and chips that he wanted and then design an experiment that you could do to test your predictions. Compare the actual results to your prediction. Explain the differences between your actual results and your actual predictions.

## Question \#3

Jerry has to take a placement test for the summer camp he is going to attend. The test has 25 multiple choice questions. Each question has four answer choices. He is certain that has 25 multiple choice questions. Each question has four answer choices. He is certain that he answered 22 of the questions correctly, but he had to guess on the other 3 questions. What are the chances that he answered all three of those questions correctly? Predict how likely it is that all of his guesses were correct and then design an experiment that you could do to test your prediction. Compare your actual results to your prediction. Explain the differences between your actual results and your predictions.

## Question \#4

Silvia is making a friendship bracelet for her best friend. She wants the bracelet to have a yellow bead, then a blue bead, then a read bead, and then a purple bead. She has put the four beads for the bracelet in a bowl. How likely is that she will pick the needs for the bracelet, in the order that she needs them, if she picks them without looking in the bowl? Predict how likely it is that she will select the right needs in the right order and then design an experiment that you could do to test your prediction. Compare your actual results in your prediction. Explain the differences between your actual results and your prediction.

## Activities Relating to Each CCSS in Learning Progression

7.SP.7.a. Activity

In this activity students will look at the shirt they are wearing today and they will determine how many buttons it has. Then, students complete the following table for all the members of the class and followed by answering the activity worksheet questions.

Procedures:
Students will first be given the "How Many Buttons?" worksheet and then complete the worksheet to complete the activity.

## Student Worksheet

## How Many Buttons?

Directions:
Look at the shirt you are wearing today and determine how many buttons it has. Then, complete the following table for all the members of your class. Afterwards, answer the activity questions below.

|  | No Buttons | One or Two Buttons | Three or Four Buttons | More Than Four Buttons |
| :--- | :--- | :--- | :--- | :--- |
| Male |  |  |  |  |
| Female |  |  |  |  |

Activity Questions:
Suppose each student writes his or her name on an index card, and one card is selected randomly.
2. What is the probability that the student whose card is selected is wearing a shirt with no buttons?
3. What is the probability that the student whose card is selected is female and is wearing a shirt with two or fewer buttons?

Solutions:
*Table values will vary depending on the composition of the class, however, using the following letters to represent the counts in each cell your students' answers should be based on the following structure.

|  | No Buttons | One or Two Buttons | Three or Four Buttons | More Than Four Buttons |
| :--- | :---: | :---: | :---: | :---: |
| Male | p | q | r | s |
| Female | t | u | v | w |

$\mathrm{P}($ no buttons $)=\mathrm{p}+\mathrm{t} /$ total of all cells $=\mathrm{p}+\mathrm{t} / \mathrm{p}+\mathrm{q}+\mathrm{r}+\mathrm{s}+\mathrm{t}+\mathrm{u}+\mathrm{v}+\mathrm{w}$
$\mathrm{P}($ Female and two or fewer buttons $)=\mathrm{t}+\mathrm{u} /$ total of all cells $=\mathrm{t}+\mathrm{u} / \mathrm{p}+\mathrm{q}+\mathrm{r}+\mathrm{s}+\mathrm{t}+\mathrm{u}+\mathrm{v}+\mathrm{w}$

## Activities Relating to Each CCSS in Learning Progression

## ${ }^{7 . S P} .{ }^{8}$ Red, Green, or Blue? Game:

In this game students will be placed into pairs and will play the "Red, Green, or Blue" Game where first, students will cut out and tape together full page versions of the dice (included on the next page). Then, students will use the worksheet as a guide to see a picture and table representations of three different than regular six-sided dice and then they answer the questions below pertaining to ununiformed probability.

## Procedures:

Decide the pairing of your students before the activity. Materials you will need for this activity is: a worksheet for each student, a full page of the dice (printed in color) for each pair, scissors, and tape. After you hand out the materials, students should be able to proceed to doing the activity following the worksheet.

## Red, Green, or Blue? Game

Directions:
After you are put into your pair group, make sure to grab a pair of scissors and tape. Then proceed to cutting out your dice individually and putting them together with tape. Before you start the game please realize that: These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other, where the number of dots on each side is shown by the table and picture below.

| Red | 3 | 3 | 3 | 3 | 3 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Green | 2 | 2 | 2 | 5 | 5 | 5 |
| Blue | 1 | 4 | 4 | 4 | 4 | 4 |



How to Play the Game:
To play the game, each person picks one of the three dice. However, they have to pick different colors.
The two players both roll their dice. The highest number wins the round.
The players roll their dice 30 times, keeping track of who wins each round.
Whoever has won the greatest number of rounds after 30 rolls wins the game.
Who is more likely to win when a person with the red die plays against a person with the green die? What about green vs. blue? What about blue vs. red? Would you rather be the first person to pick a die or the second person? Explain.

| Trial | Die Chosen by Player A | Die Chosen by Player B | Who won the round? |
| :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| 13 |  |  |  |
| 14 |  |  |  |
| 15 |  |  |  |
| 16 |  |  |  |
| 17 |  |  |  |
| 18 |  |  |  |
| 19 |  |  |  |
| 20 |  |  |  |
| 21 |  |  |  |
| 22 |  |  |  |
| 23 |  |  |  |
| 24 |  |  |  |
| 25 |  |  |  |
| 26 |  |  |  |
| 27 |  |  |  |
| 28 |  |  |  |
| 29 |  |  |  |
| 30 |  |  |  |

## Activities Relating to Each CCSS in Learning Progression

${ }^{7 . S P .8}$ Red, Green, or Blue? Game Solution:

Here are three tables that are color-coded to see who will win which rolls in each of the three possible pairings:


Description of Tables:
When red plays green, the probability that red will win is $21 / 36 \approx 0.58$
*So, red is more likely to beat green after playing many rounds.
Likewise, when green plays blue, the probability that green will win is $21 / 36 \approx 0.5$
So, green is more likely to beat blue after playing many rounds.
Similarly, when red plays blue, the probability that blue will win is $25 / 36 \approx 0.69$
*So, blue is more likely to beat red after playing many rounds.
So, it is better to choose your die second because if your opponent goes first, you can always choose a die that is more likely to beat the die he or she chose. For example, if your opponent chooses red, you can choose blue. Or, if your opponent chooses blue, you can choose green. And if your opponent chooses green, you can choose red.

Activities Relating to Each CCSS in Learning Progression
${ }^{7 . S P .8}$ Red, Green, or Blue? Game Dice:


## Activities Relating to Each CCSS in Learning Progression

7.SP.C.8.b. Activity:

Procedures:
Student Worksheet

## Mixed-up Candy Land!

Directions:
You're playing a mixed up version of Candy Land where all the characters and their treats are out of order. In this game you get to choose your character and the treat you want to use to play the game. The grid below shows all of the possible combinations.

|  | Princess <br> Lolly | Lord <br> Licorice | Princess <br> Frostine | Mr. Mint |
| :--- | :--- | :--- | :--- | :--- |
| Lollipop | Princess <br> Lolly Lol- <br> lipop | Lord Lico- <br> rice Lolli- <br> pop | Princess <br> Frostine <br> Lollipop | Mr. Mint <br> Lollipop |
|  | Princess <br> Lolly Lic- <br> orice | Lord Lico- <br> rice Lico- <br> rice | Princess <br> Frostine <br> Licorice | Mr. Mint <br> Licorice |
| Ice <br> Cream | Princess <br> Lolly Ice <br> Cream | Lord Lico- <br> rice Lico- <br> rice | Princess <br> Frostine <br> Ice Cream | Mr. Mint <br> Ice <br> Cream |
| Candy <br> Cane | Princess <br> Lolly <br> Candy <br> Cane | Lord Lico- <br> rice Candy <br> Cane | Princess <br> Frostine <br> Candy <br> Cane | Mr. Mint <br> Candy <br> Cane |



If you randomly choose a character and their candy, what is the probability that you won't be Lord Licorice or use a Lollipop?

[^1]
[^0]:    ${ }^{\text {7.SP.8.c. }}$ Design and use a simulation to generate frequencies for compound events.

[^1]:    Solution:
    There are a total of 16 possible outcomes. The probability(not being Lord Licorice)=12/16=3/4 .
    And, the probability (of not using a Lollipop) $=12 / 16=3 / 4$. Probability(won't be Lord Licorice or use a Lollipop) $=3 / 4 * 3 / 4=9 / 16$

