

High School, Algebra: Arithmetic with Polynomials & Rational Expressions

Overview

This learning progression focuses on an Applied Algebra class at Ellensburg High School in Ellensburg, Washington. In this class, there is no curriculum. Its main focus is to improve the students' applicable math skills. Therefore, these are replacement lessons. This learning progression will focus on the arithmetic and other mathematical processes involved with polynomials and binomials. The students will be doing their work in notebooks that will be collected at the end of each day. Students will be expected to take notes and answer the warm-up and exit problem in their notebooks where they will be graded on effort and ability. The following are the standards that will be covered throughout this learning progression:

- Perform arithmetic operations on polynomials.
**CCSS.MATH.CONTENT.HSA.A
PR.A.1**
- Understand the relationship between zeros and factors of polynomials.
**CCSS.MATH.CONTENT.HSA.A
PR.B.2
CCSS.MATH.CONTENT.HSA.A
PR.B.3**

- Use polynomial identities to solve problems.
**CCSS.MATH.CONTENT.HSA.A
PR.C.4**
**CCSS.MATH.CONTENT.HSA.A
PR.C.5**
- Rewrite rational expressions.
**CCSS.MATH.CONTENT.HSA.A
PR.D.6**
**CCSS.MATH.CONTENT.HSA.A
PR.D.7**

The lessons given in this learning progression will all have similar structure. Each day will begin with a warm-up questions that will either get the students thinking about the upcoming lesson or to have them recall things learned the previous day. Following the warm-up problem will be notes and opportunities for students to show their learning through worksheets, group work, board work, and/or verbally demonstrating their understanding.

Polynomial Arithmetic

Students have been doing arithmetic, including the associative, commutative, and the distributive property. This lesson is to get the students to understand that these properties transfer to polynomials. The standard that will be met in this lesson is “understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.” This lesson is to establish a good understanding of polynomial arithmetic for future problems that are more complex.

For this lesson, the students will have an entry task, called a “warm-up problem” that will be completed at the beginning of class in their notebooks. The problems on the right are an example of some warm-up problems that students will do in their notebooks. These problems are to get the students mentally prepared for the work done and concepts to be taught in this lesson.

Once the students have completed these problems, we will begin to move towards learning new material. As mentioned earlier, the students have been doing arithmetic in their previous math classes and know the properties associated with said arithmetic. Using “CCSS.MATH.PRACTICE.MP3: Construct viable arguments and critique the reasoning of others”, the students will discuss why these properties hold true for polynomials. Also, the students will be copying down notes and example problems in their notebooks. The taking of notes will be beneficial for the students because they are allowed to use their notes when completing quizzes and other assignments. Many of the students in this class have trouble retaining information due to a lack of focus. In-class discussion and the requirement of taking notes provides the students with a variety of sources for information and can be referred back to later on in the lessons. On the right is one example that can be done with using polynomials.

$$\begin{aligned}(5 \times 7) \times 3 &= 5 \times (7 \times 3) \\ 35 \times 3 &= 5 \times 21 \\ 105 &= 105\end{aligned}$$

$$\begin{array}{r} x \quad +3 \quad \left| \begin{array}{r} x \quad +2 \\ \hline x^2 \quad +5x \quad +10 \\ -x^2 \quad -3x \\ \hline 2x \quad +10 \\ -2x \quad -6 \\ \hline 4 \end{array} \right. \end{array}$$

At the end of the day, the students must complete an exit problem that looks similar to the example problems that were done in class. They will also be assigned homework that has them practice more problems and further improve their skills.

Factoring Polynomials

Once the students have shown proficiency in polynomial arithmetic, we will move towards factoring polynomials. Some of the polynomial arithmetic has led to the idea of factoring but this lesson will focus heavily on learning how to factor all polynomials. Further on in the learning progression, when the students are learning about polynomial identities, they will also be taught how to use the quadratic formula to factor specific polynomials.

To start the lesson, the students will begin the day with a warm-up problem that has them doing a long division problem so that their memory is refreshed on the process. They will also have a problem focused on factoring the coefficients of the polynomial. These problems will set the stage for what is about to come. To get the students to really challenge their thinking and understanding, we will use the practice “CCSS.MATH.PRACTICE.MP1: Make sense of problems and persevere in solving them”. The students will be asked to propose ideas how to “undo” the foiling from the previous lesson to factor polynomials based off of the two warm-up problems completed at the beginning of class. Throughout this

$$\begin{array}{r}
 x^2 + x + 3 \\
 x - 3 \overline{) x^3 - 2x^2 + 0x - 4} \\
 \underline{x^3 - 3x^2} \\
 +x^2 + 0x \\
 \underline{+x^2 - 3x} \\
 +3x - 4 \\
 \underline{+3x - 9} \\
 +5
 \end{array}$$

$$\begin{array}{r}
 x^3 - 3x + 10 \div x - 1 \\
 x^2 + x - 2 \\
 x - 1 \overline{) x^3 - 3x + 10} \\
 \underline{x^3 - x^2} \\
 x^2 - 3x + 10 \\
 \underline{x^2 - x} \\
 -2x + 10 \\
 \underline{-2x + 2} \\
 8 \\
 \text{Answer: } x^2 + x - 2R8
 \end{array}$$

$$\begin{array}{r}
 x^3 - 3x + 10 \div x - 1 \\
 1 \overline{) 1 \ 0 \ -3 \ 10} \\
 \underline{1 \ 1 \ -2} \\
 1 \ 1 \ -2 \ 8 \\
 \text{Answer: } x^2 + x - 2R8
 \end{array}$$

lesson, the students will be introduced to factoring polynomials via long division, synthetic division, and by analyzing. There will be multiple opportunities for group work followed by whole class demonstrations. Homework will also be assigned so that the students have opportunities to practice their skills. This lesson will be the prelude to finding the zeros of functions.

Finding Zeros

Now that the students are familiar with factoring polynomials and getting them in the form of binomials, they are ready to start finding zeros of the function.

The lesson will begin with a warm-up problem that focuses on factoring a polynomial by using both long and synthetic division. This process serves as a reengagement lesson. We will then use these factored polynomials to determine the zeros of the function. Namely, what values of “x” make this polynomial equal zero. From there, the students will use the zeros to draw a rough graph of these polynomials.

This new understanding of how to factor and how to find zeros will be a perfect time to introduce the quadratic formula to find zeros of simple and complex quadratics.

Again, notes will be taken in their notebooks and turned in at the end of class along with the exit problems. Homework will also be assigned to give the students a chance to sharpen their skills.

Now that students can factor polynomials and find the zeros, they can now use the practice

$$2x^3 + 16x^2 + 30x = 0$$

$$2x(x^2 + 8x + 15) = 0$$

$$2x(x + 3)(x + 5) = 0$$

$$2x = 0 \text{ or } x + 3 = 0 \text{ or } x + 5 = 0$$

$$x = 0, x = -3, \text{ and } x = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

“CCSS.MATH.PRACTICE.MP4 Model with mathematics” to model real life situations. Students will complete an assignment where they must factor polynomials, find the zeroes, draw a rough graph, and then explain what this information mean to a member of the community. This type of instruction is extremely important for students who want to know why it is important to learn mathematics. Modeling real life situations shows students that math is applicable outside of school.

Quiz

This quiz will cover the material learned in this learning progression. This quiz will serve as a benchmark assessment to establish how the students are doing thus far and what needs improvement. The topics will cover, polynomial arithmetic, factoring, finding zeros, and rough graphs.

Polynomial Identities

This is the final lesson in the learning progression. It will serve as strengthening the students' understanding of polynomials and all of the mathematics involved.

Students will be introduced to the difference of squares and how to identify these polynomials. Also, the sum and difference of cubes will be introduced as the students become more comfortable with the difference of squares. Finally, the quadratic formula will be used again in this lesson because of its vital importance in future mathematics. It is extremely important that students know when

$$a^2 - b^2 = (a - b) \times (a + b)$$

$$\text{Pr oof} : (a - b) \times (a + b) =$$

$$a^2 - ab + ab - b^2 =$$

$$a^2 - b^2$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

and how to use the formula for future mathematical success.

Again, the lesson will begin with warm-up problems, followed by notes, group work, whole-class explanations, exit problems, and homework.

Unit Test

Following the lessons within the learning progression, the students will complete a unit test. This test will be similar to the quiz with the addition of the polynomial identities. The test will also have more problems that are more complex and require more critical thinking. There will be one question at the end that the students are unfamiliar with but can be answered by using all of the knowledge they have obtained throughout the learning progression. This problem will be an extra credit problem that allows struggling students to earn some extra points and also allows students who need more of a challenge to be given that challenge. Based on the results of the test, whether the students, on average, do better than 75%, we can determine whether or not the class can move on or we need to practice a few more things.