CCSS.MATH.CONTENT.8.G.B.7 I can identify a right triangle!

**Alignment to Content Standards:** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

**Tasks:**

The following rectangle, measuring 15 inches by 12 inches, has been divided into four triangles. The figure below has been filled in with a few other dimensions.

12 in

11 in

15 in

17 in

4 in

8 in

4 in

There are three right triangles surrounding a fourth shaded triangle. Using the Pythagorean Theorem

1. Find the missing lengths in the diagram
2. Prove whether the shaded triangle is a right triangle or not.

**Commentary:**

The goal of this task is to use the Pythagorean Theorem and the given information to find the missing lengths and determine whether the shaded region is a right triangle. The students will identify the right triangles in the diagram. They will then use the lengths already given, plug them into the Pythagorean Theorem, $a^{2}+b^{2}=c^{2}$, and find the missing the lengths. Once students have solved and recorded all the missing lengths the teacher may need to give some guidance on how to use the information, along with their knowledge of the Pythagorean Theorem for part 2. Possible leading questions the teacher could ask: Can you assign the lengths of the shaded triangle a letter in your theorem?; What do you know about the longest length?; Which length has to be *c*?; How can you use those three lengths and the theorem to prove if it is a right triangle? The key insight of this task is for students to understand how they can use the Pythagorean Theorem to find missing lengths, but also as a way to prove whether a triangle is a right triangle or not.

**Solution:**

1. We first find the missing sides, focusing on one triangle at a time, and using the Pythagorean Theorem.

12 in

11 in

$\sqrt{265} $in

$$12^{2}+11^{2}=c^{2}$$

$$144+121=c^{2}$$

$$265=c^{2}$$

$$\sqrt{265 }in=c$$

4 in

$$\sqrt{32} in$$

4 in

$$4^{2}+4^{2}=c^{2}$$

$$16+16=c^{2}$$

$$32=c^{2}$$

$$\sqrt{32} in=c$$

12 in

$$\sqrt{265} in$$

 11 in

15 in

17 in

4 in

$\sqrt{32}$ in

8

4

1. We then need to prove whether or not the shaded triangle is a right triangle. In solution part 1 we expressed the side lengths as square roots. It is not necessary to take square roots to find the hypotenuses because it is only the squares of the hypotenuses that we really need to decide whether or not the shaded triangle is a right triangle or not.

Since we already found the lengths of the sides, we just need to remove or ignore the square root for this proof.

We do however need to find the square for the third side before we can proceed.

$$17^{2}=289$$

The squares of the side lengths of the shaded triangle are: 32, 265, and 289.

If the shaded triangle is in fact a right triangle, then the squares of the side lengths must satisfy the Pythagorean Theorem. The longest side length is 289, making it the hypotenuse. Therefore, the other two shorter sides, when added together, must equal 289.

$$32+265=297\ne 289$$

Since the two shorter sides do not equal the longest side, 289, the shades triangle is not a right triangle.