Title (primary CCSS Math with Title)

F.LE Patterns to Quadratic Equations

Alignment to Content Standards

[CCSS.MATH.CONTENT.HSF.LE.A.2](http://www.corestandards.org/Math/Content/HSF/LE/A/2/)

CCSS.MATH.CONTENT.HSA.CED.A.1

Tasks

Generalize the pattern by finding an explicit quadratic equation for the number of shapes that make up any given term, *n,* in the sequence. Let *Tn* represent the number of shapes that make up the *nth* term. Show your reasoning, then find *Tn* forthe next term, then draw it to check your answer.

1. n=1 n=2 n=3

2. n=1 n=2 n=3 n=4

Commentary

This task can be used as a transition/discovery project between segments about linear equations and quadratic equations. Start off by introducing number sequences and visual representations that can be modeled linearly. Moving to these more complex sequences, we will need to rely more on the visual representations to find the patterns. It is necessary in this assignment to make observations about the sequences, and may be best to work in groups to compile their observations. We are then asked to take our observations and relate them to our variable *n*. There are multiple ways to interpret both problems, but all interpretations lead to the same quadratic equations. To show that the equations created work, we use it to predict the next term in the sequence, then draw the next term based on the pattern to confirm that our equation works.

Solutions

1. *n*= 1 2 3

Students may observe that each term is made of a square with some shapes added to the sides.

$$T\_{n=1}=1^{2}+2 T\_{n=2}=2^{2}+4 T\_{n=3}=3^{2}+6$$

Relate the size of the square to *n*.

$$T\_{n=1}=n^{2}+2 T\_{n=2}=n^{2}+4 T\_{n=3}=n^{2}+6$$

Notice that the constant is always twice *n*.

$$T\_{n}=n^{2}+2n $$

Students may also observe that each term is a square missing one shape.

$$T\_{n=1}=2^{2}-1 T\_{n=2}=3^{2}-1 T\_{n=3}=4^{2}-1$$

Relating the changing size of the square back to *n*:

$$T\_{n}=\left(n+1\right)^{2}-1$$

Simplifying,

$$T\_{n}=n^{2}+2n$$

Solve for *n*=4

$$T\_{n}=(4)^{2}+2\left(4\right)=24 shapes$$

2. *n*=1 2 3 4

First notice that each term is a rectangle with two extra shapes.

$$T\_{n=1}=\left(3×1\right)+2 T\_{n=2}=\left(4×2\right)+2 T\_{n=3}=\left(5×3\right)+2 T\_{n=4}=\left(6×4\right)+2$$

Then observe that the width of the rectangle in each term is equal to *n.*

$$T\_{n=1}=\left(3×n\right)+2 T\_{n=2}=\left(4×n\right)+2 T\_{n=3}=\left(5×n\right)+2 T\_{n=4}=\left(6×n\right)+2 $$

Then notice that the length of the rectangle is two more than *n* each time.

$$T\_{n}=\left[\left(n+2\right)×n\right]+2$$

Simplifying,

$$T\_{n}=n^{2}+2n+2 $$

Solve for n=5

$$T\_{10}=(5)^{2}+2\left(5\right)+2=37 shapes$$