Making HATS [HSA.REI.B.4](http://www.corestandards.org/Math/Content/HSA/REI/B/4/)

Alignment to Content Standards

* [CCSS.MATH.CONTENT.HSA.REI.B.4](http://www.corestandards.org/Math/Content/HSA/REI/B/4/) Solve quadratic equations in one variable.
* CCSS.MATH.CONTENT. HSF-IF C.8 Interpreting Functions: Analyze functions using different representations.
* CCSS.MATH.CONTENT.8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Tasks

**The cross section of a hat can be modeled by the following polynomial function:**

$y=\frac{1}{27}(x-15)(x+15)$  **Where *x* and *y* are measured in cm.**

**Given these facts, answer the following questions and be sure to show your work:**

*Note: sketch graph in your work.*

1. How wide is the hat (not including the bill)? *Hint: we are looking for the diameter.*
2. Measurement of the Crown? *Hint: Think of the lowest point in a parabola.*

Button



Bill

Crown

Panels

Commentary

This tasks evaluates the student’s ability to discover the properties of a quadratic equation and where they can be used in the real world. The student will need to know the zero property and know how to find the vertex. This problem will be a summative assessment after the chapter examining the zero property and the process to find a vertex. Students will have to draw or solve the quadratic equation. By graphing the equation students will be able to see the quadratic formula and solve the equation. Students will be able to think critically about the problem. The problem will be done in class in pairs. It will the formative assessment. Students may have trouble solving this equation if they do not understand the quadratic equations and how to manipulate them. This problem will potentially get students to think how machine are programmed to make hats. It is real world application.

Solution

1. **How wide is the hat (not including the bill)?**
* Find the x-intercepts:
	+ Graph the Equation: $y=\frac{1}{27}(x-15)(x+15)$



We can see that the cross section is 30cm on the graph because it takes *15cm* to get to zero (*-15 to 0*) and *15*cm (*0 to 15*)

* + Using the Zero Property: Set *y=0*

$0=\frac{1}{27}(x-15)(x+15)$

$0=(x-15)(x+15)$ $0=x^{2}-225$

$0=(x-15)$ $0=(x+15)$ $225=x^{2}$

$x=15$ $x=-15$ $\sqrt{225}=\sqrt{x^{2}}$

 $x=\pm 15$

 Note: The distance between 15 and -15 is 30, therefore the cap is 30*cm* wide.

1. **Measurement of the Crown?**
* Graphing the quadratic equation on your calculator an finding the vertex and the *y* value show how big the crown is. **The Crown is 8.3cm deep because a measurement cannot be negative.**



* Plug in values to the function and make a table: (and since we know the *x* value of the vertex we look for that one in the table)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x* | 3 | 2 | 1 | 0 |
| *y* | -8 | -8.19 | -8.2 | -8.3 |

**The Crown is 8.3cm deep because a measurement cannot be negative.**

* Find the *x* value of the vertex, then plug into a table
	+ Put the equation in standard form: We will use the *a*, *b* and *c* values from the quadratic standard form $ax^{2}+bx+c=0$

$y=\frac{1}{27}(x-15)(x+15)$

$$y=\frac{1}{27}(x^{2}-225)$$

$$y=\frac{1}{27}x^{2}-8.3 a=\frac{1}{27} b=0 c=-8.3$$

We will use the formula to find the axis of symmetry, to find the *x* value of the vertex.

$$Formula for the Axis of Symmetry:x=-\frac{b}{2a}$$

$$x=-\frac{0}{2\left(\frac{1}{27}\right)} x=0$$

$$Plug back into the equation:$$

$$y=\frac{1}{27}\left(0\right)^{2}-8.3 y=-8.3$$

**The Crown is 8.3cm deep because a measurement cannot be negative.**