High School: Functions
Interpreting Functions

This learning progression will be applied in an Algebra I classroom where most students are in the 10th grade. This classroom has access to CORD's **Algebra 1: Learning in Context** digital textbook. The common core state standards aligned with this learning progression are HSF.IF.A.1 and HSF.IF.A.2. This learning progression is also aligned to the following mathematical practices: MP1, MP6, and MP7.

This learning progression incorporates the mathematical best practice of using a variety of continuous assessments that are designed to not only measure student understanding but also teacher effectiveness. From these assessments, the teacher will be able to tell which lessons resonated with the students and which fell flat and need to be retaught. Using assessment in this way reinforces that assessment is more than just a way for teachers to grade students; it is an integral part of teaching.

Students were briefly introduced to functions a week prior and have been working with linear lines in various forms, with an emphasis on slope-intercept form $y=mx+b$. Through this linear progression students will develop an understanding of what functions are, how to determine if a relation is a function, how to identify their domain and range, how to evaluate functions, how to combine standard function types using arithmetic operations, and how to evaluate composite functions.

Interpreting Functions: Understand the concept of a function and use function notation.

In order for students to understand the concept of functions, they will first be introduced to relations, which are sets of ordered pairs. Relations will be used as a stepping stone to introduce students to the concepts of domain and range and. So students will be given sets of ordered pairs and will be asked to determine the domain and range of the set and then determine the rule that defines the relation. Students will then move on to analyzing patterns in sequences of numbers. Once students have mastered this process, they will be ready to move on to functions. They will find that the difference between a relation and a function is that a function is a set of ordered pairs (x,y) such that for any value of x, there is one and only one value of y. Students will then be given sets of ordered pairs and will be asked to determine if the set of ordered pairs is in fact a function and then find its domain and range. Once students are comfortable with determining if sets of ordered pairs are functions, they will create functions to model real world scenarios and evaluate said functions.

This learning progression will start with students learning the definition of a relation. A relation is a set of ordered pairs.

Students are familiar with the term coordinate. Both ordered pairs and coordinates have the form (x,y), but they can have slightly different meanings depending on the context. Being able to comprehend the difference and when it is appropriate to use one term over the other is a way for students to attend to precision.[MP6] The initial example problem that students will be given to introduce them to the concept of a relation is having students use ordered pairs to represent the data for a receding thunderstorm using the fact that there are three seconds between the flash and sound when the storm is 1 kilometer away and you can hear storms up to 4 kilometers away. The ordered pairs will have the following form: (seconds between flash and sound, distance of storm). So the first ordered pair is (3,1) and the relation is R = {(3,1), (6,2), (9,3), (12,4)}. Students will then be introduced domain (the set of all first components of the relation) and range (the set of all second components of the relation). Thus, our domain = {3, 6, 9, 12} and range = {1, 2, 3, 4}. This will lead to the next topic of patterns.

When discussing patterns, students will first be given a sequence of numbers such as 2, 5, 8, 11 and will have to write a rule that that gives the relationship between the numbers and then find then use the rule to find the next three numbers. This will require students to apply their recently acquired knowledge in order to write the pattern as a set of ordered pairs. They will then be able to use the set of ordered pairs to make a table and/or graph, determine the slope between two points and the *y*-intercept, and then use that information to create an equation that expresses the relationship between the numbers in the pattern. [MP7] Patterns have structure and this lesson helps students build their pattern recognition skills. At the same time, students will also be building their ability to take a pattern and extrapolate the succeeding and preceding terms in that pattern thus enabling them to find and make use of pattern structure.

Students have spent the previous weeks working with linear equations. For the problem of the sequence 2, 5, 8, and 11 it will be up to the students to decide how they want to proceed when they have to create an equation to represent the relationship between the numbers in the pattern.[MP1] There are a few ways they can go about this. One way is to use the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the slope (which is 3 in this case) and then use the point-slope form to create the equation (which will be $(y - 2) = 3(x - 1)$ and when simplified is $y = 3x - 1$). Another way is to look at the numbers in the sequence and find the common difference between the terms. This will be the slope.

MP6- Attend to precision.

MP7- Look for and make use of structure.

MP1- Make sense of problems and persevere in solving them.

Benchmark Assessment Day 1: This will be done at the end of class as the exit slip. Students will be given the sequence 2, 6, 10, 14. They will have to write a rule that that gives the relationship between the numbers and then find then use the rule to find the next three numbers. Students will also state the domain and range of the relation. They will do this on note cards, which the teacher will collect and then sort through looking for common mistakes and misconceptions. The next day at the beginning of class, they will then discuss the mistakes and misconceptions that were apparent throughout the class' answers.

Students can then reason mathematically that since the first term in the sequence is 2 and the sequence increases by 3 each term that the term that proceeds the first term of the sequence must be -1, which is the *y*-intercept. The student can then use this information to build an equation that is in slope-intercept form (which will be $y = 3x - 1$). Once they have created the equation, they can then use it to find the next three terms in the sequence. Allowing students to decide how they want to approach having to build a linear equation to represent the pattern shows them that there can be more than one way to approach and solve a problem. It encourages them to use mathematical reasoning and problem solving.

The next day, to get students thinking about domain, range, and relations students will be given five sets of ordered pairs such as {(1,2), (2,3), (3, 5), (4,8)} and will be asked to state the domain and the range of the relation as a warm up. The teacher will then lead a short lesson on what makes a function different from a relation. The teacher will stress that like a relation, a function is a set of ordered pairs. However, with functions, for any value of *x*, there is exactly *one* value of *y*, which is not always the case with relations. Students will then be asked to refer back to the five relations that they were given at the beginning of class and will be asked to determine if they are functions. Students will work on these problems in groups of four. Working in groups is a mathematical best practice that gives students more opportunities to voice their thoughts and cognitive processes. It is also beneficial for students who are more timid to talk in front of the whole class. When students work in groups they can gain deeper levels of understanding and new ways to approach problems.

To solidify the idea that functions have only one *y*-value for each *x*-value, the students will be asked to plot the sets of ordered pairs. They will then analyze the plot graphs using the vertical line test. If the line covers two or more points on the plot, then it is not a function. This method gives students another way to analyze whether a set of ordered pairs is a function, namely through a pictorial method. Students previously had a symbolic method for determining whether a set of ordered pairs is a function by looking at the set and seeing if any of the entries had the same input but different outputs.

Next as a class, students will create a set of ordered pairs based off of the length of their names. The *x*-value will be the number of letters in their first name and the *y*-value will be the number of

Benchmark Assessment Day 2: This will be the exit task for the day. Students will be asked to domain and range of the relation they created based on the letters in their names and determine whether it is a function and explain why it is or is not in 1- 2 sentences. This falls under the mathematical best practice of writing about mathematics.

letters in their last name. For example John Smith would be (4,5). For the exit task, students will be asked to find the domain and range of the relation they created and determine whether it is a function. The teacher use the students' answers on their exit tasks to assess their understanding of domain and range and functions [HSF.IF.A.1]

On the third day, students will learn to create functions to model real world scenarios and evaluate said functions. Students will be separated into groups of four and will then be asked to create a function that models the following scenario: You are saving up for a trip to Madagascar that costs $6,000. You find $500 on the street and use it to start your savings for the trip. You decide that you can put $100 into your savings a week. Create a function to model your savings. How much money have you saved after 12 weeks? How many weeks will it take for you to save enough money for the trip? What is the domain of your function? What is the range? [HSF.IF.A.2]

This question is designed to incorporate the previous two days' material in with the notion of function notation. It is a sort of culmination of the previous lessons. In order to successfully complete this problem, students will need to understand first what a function is, how to interpret the scenario so that they are able to write a relation to help model what is happening and then use it to write a rule that they can turn into their function, and how to determine the domain and the range of their function.

Again by working in groups, students will have access to another resource that they might not necessarily have if working individually: their peers. They will be able to have discussions that can clear up misconceptions or misunderstandings. Group work paves the way to cooperative learning. The Benchmark Assessment for this lesson is very similar to this problem. However, students will do the Benchmark Assessment individually. Having just done a similar problem in groups, the students will be able to see how much of the process and the concepts they can recall when working on their own, thus pointing out to the students areas where they may be excelling and areas where they may be still be struggling. This will also help inform the teacher on which students have a firm grasp on the topics covered in this progression and which students still require more time and discussion.

HSF.IF.A.1- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then *f*(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation $y = f(x)$.

HSF.IF.A.2- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Benchmark Assessment Day 3: Students will be given the following question: A sunflower is 3 inches tall at week 0 and grows 2 inches a week. Create a function that models the sunflower's growth. How tall will the sunflower be at week 7? What is the domain of the function? What is the range? (HSF.IF.A.2)