**High School: Algebra**

***Reasoning with Equations and Inequalities***

This learning progression will be taught in a high school sophomore algebra classroom. The common core standards that this learning progression aligns with are HSA.REI.C.5, HSA.REI.C.6, and HSA.REI.C.7. This progression also makes use of the following three mathematical practices: MP1: Make sense of problems and persevere in solving them, MP3: Construct viable arguments and critique the reasoning of others, and MP4: Model with mathematics.

Prior to this learning progression, students will have been introduced to variables and very basic functions. This learning progression is designed to be taught directly after a unit on linear equations, and will introduce students to systems of linear equations and how to solve them. This learning progression will begin by having students use what they have learned about linear equations to model a real life problem, and then introduce students to three different methods for solving systems of linear equations. The order of the progression will allow students to build on their conceptual understanding of linear equations and then introduce a new procedure related to the original concept.

To support the learning of all students in the class, students will spend a lot of time working in small groups during this progression. Working in groups will provide a great opportunity for students who understand the material to teach their peers who might be falling behind. Peer teaching will not only help those being taught, but will reinforce concepts for the gifted students. Several types of instruction will also be used to promote learning for all students. Throughout the progression students will have an exploratory lesson, direct instruction, and a modeling activity. Students will be assessed daily and will receive quick and informative feedback as well.

***Reasoning with Equations and Inequalities***

**Solving Systems of Equations**

This learning progression will incorporate concepts and procedures that students learned in the previous unit on linear equations. The first activity for this learning progression will be an exploratory activity where students will use linear equations to model a car race. For this activity, students will work in groups to choose the best equation for a car to win a race. Students will have the option to choose equations with high slopes and low intercepts, or low slopes and high intercepts. After each group

**MP3-** Construct viable arguments and critique the reasoning of others

has chosen their equation, the equations will be plugged into the FluidMath App. FluidMath allows students to actually visualize a race between cars that move according to the equations that they’ve created. This activity will not only allow students to apply what they’ve learned about linear equations to a real world model, but will transition their thinking from dealing with one equation to several equations. This first activity will serve primarily as a review lesson that will smoothly transition and connect ideas to the bulk of the learning progression.

After students have begun thinking about systems of linear equations, they will be introduced to some vocabulary. Students will be introduced to terms such as parallel lines, intersection points, system of equations, etc. Once students have a grasp of what a system of linear equations is, they will learn three different methods to solve them. First they will learn how to solve linear equations by graphing them. The graphing method is very visual and should be the easiest method for students to use. To assess that students understand this method, students will work in groups to solve several systems of equations. They will then present their graphs and their mathematical thought processes to the rest of the class. Each group will be given systems that will force them to use appropriate vocabulary in their presentations. They will be given at least one system with parallel lines and they’ll have to talk about where the lines intersect.

Next students will be introduced to the substitution method. This method will force students to use some knowledge from past units, and may be a little more difficult for students. They will rely heavily on mathematical reasoning skills to learn this method. The substitution method may take longer for students to learn, and they will be given several assessments to prove that they understand how to use this method. Students will be asked several questions throughout this lesson, and will have

**HSA.REI.C.6-** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Hinge Questions** for first lesson**:** What do you notice is always true about the equations of parallel lines? Can a system of equations with equal slopes be solved? Why or why not?

**Benchmark** for lesson two: Without graphing, construct a system of two linear equations where (-2,3) is a solution to the first equation but not to the second equation, and where (5,-2) is a solution to your system.

to complete a worksheet on the substitution method. If students take longer than anticipated to grasp the procedures in this lesson, the following lesson will be pushed back a day to review this method.

After students are procedurally fluent in solving systems of equations using the substitution method, they will be introduced to the final method: the combination method. This method uses many steps and processes and may also be difficult for students to understand at first. Students should be familiar with each process needed to use this method, however combining the steps will be challenging. To assess students on this method they will also be given a worksheet to work on. Because by this point students should be rather familiar with systems of equations, this worksheet might be a little more difficult than the last. Students will not only be assessed on their ability to solve systems using the combination method, but they’ll be assessed on their ability to recognize systems of equations in word problems. Students will be given several story problems, will have to use the given information to create a system of equations, and then will have to solve the system using the combination method.

The final activity of this learning segment will be a review lesson that assesses student understanding of each of the three methods to solve systems of linear equations. Students will also be assessed on their ability to create systems of equations from word problems. For this activity students will work in three stations. At each station, students will use a different method to solve a complex word problem. Each problem will have items that students can use at the station to model the problem. For example, at one station students will be able to work with pennies and a scale to model the problem given on the right. Each station will have a similar problem, and students will not be able to move to the next station until they have correctly solved the problem at the station they begin at.

**HSA.REI.C.5-** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**MP1-** Make sense of problems and persevere in solving them

**MP4-** Model with mathematics

**Penny Problem:** A roll of pennies contains 50 coins. Some coins are from before 1938 and weigh 3.11 grams, while some were minted after and weigh 2.50 grams. If a particular roll of pennies weighs 138.42 grams, how many of the new lighter pennies and how many old heavy pennies does this roll contain?

This activity will allow students to work in groups and to peer teach each other, and those students who struggle with the concepts can learn from those who understand it well. This activity will assess student conceptual understanding, procedural fluency, mathematical reasoning, and vocabulary. Students who finish this activity early will be given one more worksheet to work on until all students have completed all three stations. This worksheet will have several problems that will transition students to thinking about systems of equations with quadratic equations as well. It will be a simple activity to get the students thinking about what will be covered in the following learning progression. Throughout this learning progression students will build on previous knowledge to learn several methods to solve systems of linear equations.

**HSA.REI.C.7-** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line *y* = -3*x* and the circle *x*2 + *y*2 = 3.