**High School Algebra 1**

***Seeing Structure in Expressions***

This learning progression will be taught in a 10th grade High School Algebra classroom and the following two Common Core State Standards will be used as goals: HSA.SSE.B.3 and HAS.SSE.B.3.a. Additionally, the mathematical practices that align with this learning progression are the following: MP1 Make sense of problems and persevere in solving them, MP5 Use appropriate tools strategically, and MP6 Attend to precision. The textbook this class uses is CORD Algebra 1 Learning in Context 4th edition. Specifically, I will be using chapter 7 on polynomial and factors.

Students have previously been introduced to and worked with linear and quadratic functions. Specifically, with linear functions they have practiced finding roots/x-intercepts and y-intercepts. In this learning progression students will be once again looking at roots, but of quadratic functions. The first skill and portion of this lesson will be to examine how to manipulate a function into a factored form so as to reveal information about the function itself. Then, next step will be to have students practice this process with a variety of functions having various leading coefficients, making sure that all given functions can be factored into binomials evenly. The third lesson in this progression will be to consider cases where the function cannot be factored into binomial terms that are easily found through guess and check, factor fish, factoring using factors of the first and last coefficients, or some other method. Thus, with the third lesson the quadratic formula will be introduced. All of these lessons will focus on finding roots and the meaning behind the zeros.

To help students gain the most from this learning progression two additional methods of instruction besides lecture and question/answer discussions will be incorporated into the lessons. The first will be using groups to help students support their own understanding and those of others. According to the study done by Gulfer Capar and Kamuran Tarim in 2015, having students work in groups provides a greater increase of student understanding and achievement and thus helps support the current theory of cooperative learning in connection with mathematics. Since student will be working and collaborating together on the mathematics, student who normally struggle in this subject area will have immediate supported, setting students at ease and allowing for focus to be on the material. The second added method will be using games to help strengthen understanding and recall skills. A study conducted at Michigan State University by James Banfield and Brad Wilkerson in 2014 found that there was an increase in student understanding and skills with the material when the instructor used games to help teach the lessons. Thus, during my learning progression I will also include time to practice the material through the use of a game.

**HSA.SSE.B.3** Choosing an producing an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

**MP1** Make sense of problems and persevere in solving them

***Write expressions in equivalent forms to solve problems***

Since students will be building on previous knowledge and understanding, they need to have a clear goal in mind to achieve new learn. Students will be reminded of the terms zeros and roots[MP1]. To help with this, I will engage the students in a discussion about the meaning of these roots by examining a quadratic function, namely $y=x^{2}-4$. By taking a closer look at this functions actual graph students will be help to see that roots and zeros of a function the point on the graph where it crosses the x-axis[MP1]. These terms will again be defined and discussed in the context of both linear and quadratic functions and this will establish the goal of mastering the new tasks of factoring a quadratic and using the quadratic function. Thus, in the first lesson students will practice rewriting the expressions into equivalent form, first with some warm up problems, and then moving onto the short lecture discussing the multiplying of binomials[HSA.SSE.B.3].

The process of multiplying binomials should be relatively familiar to the student at this point and after the short review of how to combine them the lecture will discuss taking them apart into what is called factors. I will provide several examples of how this method of factoring works using the guess and check method, which I feel is the simplest of forms for students to start learning factoring with. Then after engaging students in a question and answer discussion, I will group students into three people per group and have they work on factoring a number of problems for self-practice.

**Day One Benchmark:** explain in your own words what a factor of a polynomial function is. [Benchmark1]

**HSA.SSE.B.3** Choosing an producing an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

**MP1** Make sense of problems and persevere in solving them

**MP6** Attend to precision

For the first day assessment, I have set for a benchmark being able to explain what a factor of a quadratic is in their own words[Benchmark1][HSA.SSE.B.3] . Since this topic of factors and their meaning is essential to the subject, I want to insure that every student understands the reason and goal for working hard to factor any polynomial function, in this case specifically the quadratic[MP1]. This assessment will be given at the end of class as an exit slip, which I will collect as they leave the room.

I will use this exit slip to help me tailor my next day’s lesson to insure that, for the students, the goal of learning this new technique of factoring has a purpose. Additionally, I will include on the exit slip a place for the student to leave me remarks as to how well they understood the lesson of factoring and if they were able, while working in groups, to solve any of the problems without their partners. This too will help me get an idea of which student will possibly struggle the most on the following day’s learning progression.

Day two of the learning progression will be more involved. I will start the lesson off with a short review of the previous day’s learning goal and use this time to clarify any misunderstandings I found on the exit slips from each student. Then with two warm up problems, I will have student use a note card and write their solution to the two problems. The first problem will be to build confidence in the students by giving them three binomials to multiply together. This will both test their skills with Algebra and their understanding of changing an expression into an equivalent form[MP6]. For example, I will use $y=3\left(x-2\right)^{2}-1$ and have the student manipulate this equation back into the commonly known form of $y=ax^{2}+bx+c$, looking for the students ability to carry out the manipulation with precision. Being that this first problem is familiar to the students, it will set them at ease and help prepare them for the new task.

The second warm-up problem will be similar to the previous day’s exercises when they worked together to find the factors of an expression. However, on day two of the learning progression during the warm-up they will be working alone. Then after students have completed this warm up problem they will bring their note cards to me and I will quickly look through each and pick out a few that made common mistakes and use this as a teaching moment to help others who may have also made the same mistake. Since I have built a good report with my students they will not fear offering a wrong answer, because they know that my classroom is a safe environment and mistakes are simply learning opportunities. Thus, without letting the class know which student made the mistake I will recopy their solution and the class will explain why or how the student went wrong, thereby helping this student and others see the proper method for solving the given problems.

After answering questions from the warm-up problems and righting any misconceptions, day two’s lesson will begin by focusing on the actual process of factoring[HSA.SSE.B.3.a]. As a class we will do a few more problems of increased difficulty using the guess and check method. I will again group students into threes and I will have them factor several more difficult problems on their own, making my way around the room observing and helping students[MP6]. Similar to the above example of $y=3\left(x-2\right)^{2}-1$, I will be watching the students factor each equation checking that the answers they have will still be equivalent after multiplying the binomials together. When I deem the students ready to learn a more effective factoring methods I will bring the class back to focus at the front and lecture with a question and answer discussion about better ways to factor problems other than guess and check.

I will teach the students the factor fish method, showing the class several example that they will add to their notes and use as a guild to the next small set of expressions for them to factor. They will return to the same groups and solve each problem with this new method and then check they factored correctly by using the first guess and check method to evaluate themselves[MP6]. This continuous checking of their own work and of those around them will help each student become more accurate with manipulations to lessen dropped negative signs or incorrect multiplying.

**Day Two Benchmark:** I can factor a quadratic function [Benchmark2]

**HSA.SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.

**MP6** Attend to precision

**MP5** Use appropriate tools strategically

The last two activities for the day will be as follows: a benchmark assessment and a practice game.

Once students have finished the small set of problems using the factor fish method, I will have them return to their seats for a small exit question quiz. This quiz will serve as a benchmark assessment of their understanding to the day’s material on actual factoring of a quadratic function[Benchmark2]. Their goal is to factor three problems that increase in difficulty. The students are free to use either method for solving the problems, however, they must use the factor fish method with one of the problems simply to show understanding of the tools and strategies available[MP5]. For example, one of the problems presented, $y=2x^{2}+5x-7$, looks to more difficult having a leading coefficient of 2 and a -7. Thus, students may struggle to factor using the guess and check, but find that the factor fish method will present the answer simply as they fill out the boxes.

After assessing the initial understanding with the benchmark assessment, I will split the class into three groups evenly. They will form three lines towards the front of the room and will face the front white board for the purpose of playing a game for practice. The three teams will compete against each other for points by coming to the board three at a time and factoring as quickly and accurately as possible the problems that I will verbally read out loud[MP6]. The first person on each round to correctly factor the problem gains a point for their team. Since points can only be earned for correct answers I will be watching that negative signs be placed properly. For example, one the of the previous problems above, $=2x^{2}+5x-7$, I will use again to insure that the factored form is precise looking for (2x+7)(x-1) not (2x-7)(x+1). They are free to choose either method, guess-and-check or factor fish, to solve the problem. If a student does not know what to do he can quietly discuss how to do the problem with his teammates only after attempting it himself first[MP1]. Thus, by allowing discussions between teammates student can hear and share knowledge form alternate sources, strengthening both parties understanding.

**HSA.SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.

**MP1** Make sense of problems and persevere in solving them

**MP5** Use appropriate tools strategically

**MP6** Attend to precision

The purpose of the game is to allow more practice for the students and at the same time making the learning progression interesting and fun. As was mentioned above there is a greater chance of students learning and remembering material if it is done so using a game. Therefore, the competition offers some pressure and requires students to recall and ingrain in themselves the process of factoring a quadratic.

**HSA.SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.

**MP1** Make sense of problems and persevere in solving them

**MP5** Use appropriate tools strategically

**MP6** Attend to precision

Day three of the learning progression will start with another small set of warm-up problem for the students to factor and again they will answer each on a note card. After I quickly look through the notes cards and again pick out my favorite mistakes and discuss these with the class[MP6], I will pose a question to them:

‘What if you cannot factor the quadratic, how will you then find the roots of the polynomial?’

This will lead into the discussion of the many quadratics that are not easily factored and how to handle them, thereby insuring that the tools they have learned thus far have great value[MP5]. I will introduce the quadratic formula and reiterate the goal of learning to find roots or zeros of a function. I will not go into detail just yet as to where the quadratic formula comes from, but rather will focus on the use of the formula.

I will give several examples of when to use the formula and workout each problem carefully to explain possible traps students will fall into[MP6]. For example, if the terms are not in descending degree order then the *A, B,* and *C* values will not be the first term they come to reading left to right. Additionally, since the formula uses –*B*, helping students understand this mean the opposite of whatever the *B* value currently is. Then once the examples have been shown and the lecture/discussion finished, I will spend the remaining part of the class having students practice on a worksheet using the quadratic formula with several problems[HSA.SSE.B.3.a].

The last portion of the class period will focus on overall student understanding of the three lessons presented in the learning progression. I will use a third benchmark assessment to examine all of the content we have discussed. The assessment has the purpose of seeing where students are struggling and what misconceptions they are having about factoring a quadratic function or expression[MP1]. Since several of the problems will be difficult like, $y=3x^{2}-15x+57$, I will allow the student to use calculators so that the struggle to learn this new concept and formula will not be too great so as to hinder progression of learning. They should struggle some, since only through some amount of struggle do we learn, but it will not be beyond their ability.

This assessment will be looking for two key ideas of understanding from each student: what does a factor mean or tell you about the function?[HSA.SSE.B.3.a] And how do you know which method to use when trying to factor a polynomial, specifically a quadratic?[MP5]

This benchmark assessment will include a question having the students explain the meaning of factors and their purpose[HSA.SSE.B.3]. It will also ask which method to use given a quadratic and why the student chose this method for solving. Lastly, it will have the students demonstrate all three methods for factoring: guess and check, factor fish, quadratic formula, and will check for precision by expecting student to show all work[MP6]. Factor fish method, also showing that it works by multiplying out their answer. And lastly, using the quadratic formula to solve a problem that cannot be factored easily like $y=x^{2}-4x-11$[MP5].

All three lessons in this learning progression connect together to help student understand the importance of factoring a quadratic and how to do so. The progression will introduce new topics every day as well as reinforce previous discussions. Thus, student will be continuing to build upon previous knowledge to help meet the two common core standards mentioned above as well as exercise their understanding with the mathematical practices. With each day’s benchmark assessment I will gain a clear picture of my students and their progress towards understanding.

**Day Three Benchmark:** I can factor a quadratic using several methods, including the quadratic formula, and understand the meaning of a factor

**HSA.SSE.B.3** Choosing an producing an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

**HSA.SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.

**MP1** Make sense of problems and persevere in solving them

**MP5** Use appropriate tools strategically

**MP6** Attend to precision