**High School Functions**

**Interpreting Functions**

This learning progression will be applied to a 12th grade High School classroom and the Common Core State Standards that are aligned to this progression are in the “Understand the concept of a function and use function notation” cluster and include: CCSS.MATH.CONTENT.HSF.IF.A.1, CCSS.MATH.CONTENT.HSF.IF.A.2, and CCSS.MATH.CONTENT.HSF.IF.A.3.

Students have had brief experience with functions in their past classes and earlier this year. This progression will take this experience and give the students a deeper level of understanding of functions. They will take their general knowledge and focus on the domain and range of functions and how this correlates to the independent and dependent variables of the function. Then students will engage their procedural understanding and calculation skills to use an input to find an output. The students will then generalize and abstract to understand that sequences can be thought to act like functions and can be modeled similarly to normal functions.

To foster better learning and peer tutoring, the students will be exposed to a mix of cooperative learning and ability grouping. Students who are exceling, who are meeting expectations, and who are no meeting expectations will be grouped in equal amounts. This will give the opportunity for struggling students to get more help that what the teacher can provide individually and will allow the students who are meeting expectation and those who are exceling to tutor the struggling student to improve their own understanding of the materials being taught. This is particularly beneficial for students with exceptionalities and those who are bilingual or multi-lingual learners. These students can be grouped with more fluent students to practice their math skills as well as practice their speech.

**Understanding the concept of a function and use the function notation**

Students will first review the concept of functions. They will be reintroduced to the graphs of functions as well as what it means for a set to be the domain or range of a function and what it means to be an independent and dependent variable. Students will first be given the definition of an independent and dependent variable. Because these definitions and concepts are very abstract, the best way to develop student understanding is to have multiple examples. The teacher will do two to three examples of real world functions where there are clear independent and dependent variables. There will be examples of variables being dependent in certain situations and independent in others to highlight that the context of the variables in the function is important. After these examples the teacher will give the students one problem to try in small groups. Each group will present their answers to the class and then the class will discuss which answers are right and then why certain groups, if any, got the wrong answer.

Students will then be given the definitions of domain and range. The relationship between domain and range will be compared to using a vending machine. The inputs are money and the outputs are the particular snacks in the machine. In this way the students can see that there are particular inputs and particular outputs. They will be able to visualize that you cant put snacks into the vending machine to get more snacks because it only takes money. They will see a concrete example of a domain issue. They will also be able to see that you are not able to get certain things out of the machine. This is a concrete example of a range issue. The teacher will do two to three example problems of taking a given function and identifying the domain and range. The teacher will be sure to highlight the fact that the range of a function can be limited, that is not all of the real numbers. Students will then be given a problem to do individually. The students will not discuss their answers with the class but will be given the real answer and time in their small groups to discuss any issues that the group members have. Additionally the teacher model a different type of problem by giving a domain and range and ask the students give a function that could satisfy these conditions. The students will then share answers and discuss what is necessary for a function to satisfy the domain and range. The purpose of the activity is to make student more comfortable with the procedural fluency of finding domains and ranges which further improves their conceptual understanding of functions.

The Learning Targets for this section are: I can identify dependent and independent variables. I can find the domain or range of a function given a range or domain.

For the three above class activities the following will be a Benchmark Assessment. Students will at the end of class receive a half sheet of paper and be asked to provide a definition for domain and range with a function with domain and range labeled and provide a definition for independent and independent variables and give an example a specific situation with the independent and dependent variables labeled. If students are able to correctly perform the Benchmark Assessment they will have a deep enough understanding of functions for the purposes of the high school classroom. Having this basic understanding has a twofold purpose: it shows that the previous aligned with CCSS.MATH.CONTENT.HSF.IF.A.1 and it give students the ability to move onto evaluating functions and answering question about function in the context of specific problems. Understanding the vocabulary and being able to correctly identify examples of each will make it easier for students to accurately calculate the output of a function and then take the context of the domain and range to answer questions in the context of the problem.

Next students will transition to evaluating functions and then answering questions in the context of specific problems. Students will be begin by reviewing the definitions of domain, range, independent variable, and dependent variable. This will bring the students into the lesson and put the idea of input and output values fresh into their mind. They will be introduced to evaluating a function by being asked, “what happens to a number from the domain when I plug it into the function?” The teacher will then take a familiar function like f(x)=x2 and create a table of different values in the domain and show where it gets sent in the range. This is a very mechanical skill and focuses mostly on procedural understanding. So student will be placed in groups given 10 functions to evaluate at 3 different points.

Next students will take this new skill and apply it to real world problems. The teacher will put a real world problem on the board that is modeled by a function. They will then ask the question “what does it mean when I say f(3)=10?” The student will then be given 2 minutes in small groups to discuss what it means and then the class will reconvene and discuss the possible answer. The teacher will then explain what f(3)=10 means in a real world context. To practice this, students will be given 3 real world problems and then asked to determine what it means for f(4)=5. The three questions will be interrelated to show that depending on the problems context, the answer can be different. This highlights that understanding what the independent and the dependent variables are is important to real world problems.

The Learning Target for this lesson is: I will be able to use functions to solve real world word problems.

For the above activity, the following will be a Benchmark Assessment. They will be given a story problem. The story problem will have the students finding the cost to own a vehicle based on how many miles the person drives per day on average. The students will be given a function that represents the situation. They have to state the domain, the range, and then find the cost based on different average miles per day numbers. The will also need to state what f(5)=6 means in the context of the problem. This assessment will show that the students are able to use functions, function notation, and “interpret statements that use function notation in terms of a context.” Students who are able to do this show they fully understand how functions work and they understand the details of what make up functions. Also, this will show that the lesson was aligned with CCSS.MATH.CONTENT.HSF.IF.A.2. This together shows that students will be prepared to view sequences in terms of a function. They can take the different aspects of sequence and identify a dependent variable, an independent variable, and a concrete function that models it.

Next students will take their knowledge and experience regarding functions and use it to rephrase sequences to fit in the context of a function. The students will begin by reviewing how to input values into a function. Then the students will have an introduction of what a sequence. Specifically, they will be given arithmetic sequences to begin with and then geometric sequences. They will then be given sets of numbers and then asked if in that order, do then make a sequence. The students will take time in small groups to discuss this and then share what they though with the class. Then they will be asked to create two different sequences using the same starting value. They will then be asked to take the sequences they made and write a function that represents it. Next, students will be introduced to a famous sequence that is defined recursively, the Fibonacci Sequence. They will be shown how it works and how elements of the sequences are calculated. In addition, they will be shown real world examples of the Fibonacci Sequence.

For the above acidity, the following will be a Benchmark Assessment. There will be two questions that give an infinite sequence that is not defined recursively, create a function that gives the sequence as outputs. The third question ask the students to find the first 15 Fibonacci Numbers. This assessment will show that the student can use functions to generate a sequence and that they are able to use function properties to develop a function that will describe a given sequence. Being able to successfully complete the Benchmark Assessment will show that the student is able to work with functions effectively and that they are able to view sequences in terms of functions. Additionally, the Fibonacci Sequence question is simply to test their understanding of sequences that are defined recursively so they can use that skill in later mathematics to model real world situations.