**High School Geometry 1**

***Experiment with transformations in the plane***

This learning progression will be taught in a 10th grade High School Geometry classroom and the following three Common Core State Standards will be used as goals: [HSG.CO.A.3](http://www.corestandards.org/Math/Content/HSG/CO/A/3/), [HSG.CO.A.4](http://www.corestandards.org/Math/Content/HSG/CO/A/4/) and [HSG.CO.A.5](http://www.corestandards.org/Math/Content/HSG/CO/A/5/). Additionally, the mathematical practices that align with this learning progression are the following: MP5 Use appropriate tools strategically, [MP7](http://www.corestandards.org/Math/Practice/MP7/) Look for and make use of structure, and [MP8](http://www.corestandards.org/Math/Practice/MP8/) Look for and express regularity in repeated reasoning. The textbook this class uses is CORD Geometry 1 Learning in Context 4th edition. Specifically, I will be using chapter 4 on polynomial and factors.

Students have previously been introduced to patterns and relationships between two things. For example, in previous classes they have discussed series of geometric functions and more recently similarities in triangles. Specifically, students have worked with distances on the coordinate plane is relation to the origin and a point and how this constant equidistance moved around the origin would rotate a point which was a topic discussed in chapter 3. In this learning progression students will be once again looking at these rotations of points with the larger and focused picture of rotations, reflections, and translations of various geometric figures. The first skill and portion of this lesson will be to examine how to manipulate an object/shape on the coordinate plane through reflections that will maintain its original form. The various reflections, which will depend on the geometric object used at the time, will include over the y-axis, x-axis, y=x and y= -x. The students will practice this process with a variety of shapes and discover which shapes can or cannot use one of the above mentioned reflections. For example, a square can be both reflected across the y= x and y= -x, but a rectangle cannot. The second lesson will then discuss rotations of the same geometric objects and once again help students investigate which rotations certain shapes can and cannot rotated x number of degrees. The third lesson in this progression will be to consider translations of objects so as to still maintain shape, size, and angles. The third lesson will also help clarify the previous topics with practice with all of the movements as well as help students understand notation for the actions, like T(2, 3) referring to a translation of all vertices of an object to the right by 2 and vertically by 3.

To help students gain the most from this learning progression two additional methods of instruction besides lecture and question/answer discussions will be incorporated into the lessons. The first will be using groups to help students support their own understanding and those of others. According to the study done by Gulfer Capar and Kamuran Tarim in 2015, having students work in groups provides a greater increase of student understanding and achievement and thus helps support the current theory of cooperative learning in connection with mathematics. Since student will be working and collaborating together on the mathematics, student who normally struggle in this subject area will have immediate supported, setting students at ease and allowing for focus to be on the material. The second added method will be using games to help strengthen understanding and recall skills. A study conducted at Michigan State University by James Banfield and Brad Wilkerson in 2014 found that there was an increase in student understanding and skills with the material when the instructor used games to help teach the lessons. Thus, during my learning progression I will also include time to practice the material through the use of a game.

***Congruency with transformations***

Since students will be building on previous knowledge and understanding, they need to have a clear goal in mind to achieve new learning. Students will be reminded of patterns and how they can help offer structure and constancy when looking at new material. To help with this, I have planned to open the discussion of reflections with a fun puzzle that requires students to place wolves *W* and sheep *S* in a 5x5 grid is such a way that the sheep cannot be captured by the wolves[MP5, MP7]. The activity starts easy only having one sheep and two wolves, but then increases in difficulty as each round adds another sheep or wolf until the ultimate goal of five wolves and three sheep on the 5x5 grid. By taking a closer look at this fun puzzle model, students will reconnect with previous lessons on patterns and prepare themselves to thing about reflections since many of the answers student will provide will simply be a reflection of another students[MP7].

**Benchmark Assessment 1:** Write the sequence of reflections to transform the original figure to the new

[**HSG.CO.A.3**](http://www.corestandards.org/Math/Content/HSG/CO/A/3/)**:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

[**HSG.CO.A.4**](http://www.corestandards.org/Math/Content/HSG/CO/A/4/)**:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MP5:** Use appropriate tools strategically

**MP7:** Look for and make use of structure

[**MP8**](http://www.corestandards.org/Math/Practice/MP8/)**:** Look for and express regularity in repeated reasoning

At the start of this activity there will be several possible answers, however by the time the lesson has reached the ultimate goal there will be only one correct solution and any others will be a rotation or a reflection of this one single answer. After this warm-up problem has been completed, the lesson will then focus further on these reflections and as a class we will investigate each reflection learning the mathematical vocabulary gaining understanding into words such as reflection, rotation, and translation. Additionally, we will discuss how one would reflect over the y-axis, x-axis, y= x, and y= -x[HSG.CO.A.3, HSG.CO.A.4]. Once students have the basics on the differences between the vocabulary terms and the picture that matches this situation, I will place them in groups and have them solve a worksheet of problems naming each reflection with the proper notation as well as taking a word problem and providing the appropriate picture to match[MP5]. The purpose of this activity is to assess the student knowledge and understanding of the vocabulary as well as having them apply the use of these terms[MP7]. Also this activity will help students immediately become familiar with the pictures connected to the terms and what process of reflections would need to happen to go from the old picture, given, to a new one, also given, in a comparison process[MP8].

Lastly, the end of the first day’s activities will end with a benchmark assessment in the form of an exit question to check overall student understanding[Benchmark 1]. I will ask student to provide the notation and vocabulary in written form answering a question about a trapezoid starting in quadrant one and ending in quadrant three in an alternate orientation[HSG.CO.A.3]. They will need to provide their reflection process that they chose to accomplish the task. For example, if they chose to first reflect the trapezoid across the y-axis and then the x-axis they will have provided the same as another who simply reflected it across the y= -x[MP7]. By having students provide the full scheme of their actions, I can see how their minds are working and if they are understanding that reflections many need to be done in a certain way if working with a specific shape that would provide a different solution had they solved it another way[MP8, HSG.CO.A.4]. Thus, the scheme will again help me see how students are progressing in their understanding of the topic of reflections.

**[HSG.CO.A.3](http://www.corestandards.org/Math/Content/HSG/CO/A/3/):** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

[**HSG.CO.A.4**](http://www.corestandards.org/Math/Content/HSG/CO/A/4/)**:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MP5:** Use appropriate tools strategically

**MP7:** Look for and make use of structure

[**MP8**](http://www.corestandards.org/Math/Practice/MP8/)**:** Look for and express regularity in repeated reasoning

Day two will start off with a short review of day one’s actives using their exit question to help clarify any misconceptions and misunderstandings on the topic of reflections. Since students will be providing self-assessment from the day one exit slip, I can also use this time to help those students who found the reflections topic difficult, to help them more one on one or pair them with others who were able understand proficiently. This will then be the transition into day two’s topic of rotation[HSG.CO.A.3, HSG.CO.A.4]. Again I will talk about the sheep and wolves activity showing students that many of the answers provided are rotations of other answers. We will then start a new activity having students working in groups show and write out how an object or shape would look if they rotated it by a certain number of degrees. For example, one such problem will be working with a square and the worksheet will start off by having them rotate it 90 degrees, then 180, 270, 360[MP8, HSG.CO.A.4]. Students will quickly see that the orientation of the object will be exactly the same as it would be had they not rotated it. The work sheet will then ask them to explain in their own words why the rotation done to the square does not change the overall orientation of the square. Similarly, students will follow the same process through a second and third shape, triangle and rectangle/regular polygon respectively[MP8, HSG.CO.A.3].

By having students follow the same process several times, they will not only gain proficiency with the terms and actions of the rotations, but they can exercise their mental ability to think about an object abstractly[MP8]. This repeated process will develop reasoning processes in the student’s minds that they can use to help analyze each problem and each new tool they are learning throughout this lesson[MP5, MP7].

Since the day two’s activity will be only focused on rotations and learning the vocabulary that accompanies, students will be doing a lot of self-discovery through leading worksheets and discussions about situations. For example, as mentioned above they will repeat the process of rotating several objects various degrees[HSG.CO.A.4, Benchmark 2]. One such rotation discussion we will have will be about an equilateral triangle and instead of rotation it like the square and rectangle, we will need to discuss the proper number of degrees this triangle must rotation to reach an orientation similar to the starting position. This brief look into the importance of understanding that the number of sides an object has will also dictate how many degrees of rotation will have to happen in order for the object or shape to orientate itself similar to the original. Specifically, a square will need to rotate in increments of 90 where as an equilateral triangle must rotate in increments of 60 degrees[MP7, HSG.CO.A.3]. Also while discussing rotations of various degrees, I will bring back into the students minds about day one’s topic of reflect it across one of the lines of symmetry, meaning we must take into account the number of sides and object must have in order to reflect it across certain line of symmetry[HSG.CO.A.4, Benchmark 2]. Thus, tying in the current discussion with day one’s discussion.

**Benchmark Assessment 2:** Write the sequence of rotations by set number of degrees around a vertex or origin to transform the original figure to the new

[**HSG.CO.A.3**](http://www.corestandards.org/Math/Content/HSG/CO/A/3/)**:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

[**HSG.CO.A.4**](http://www.corestandards.org/Math/Content/HSG/CO/A/4/)**:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

[**HSG.CO.A.5**](http://www.corestandards.org/Math/Content/HSG/CO/A/5/)**:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**MP5:** Use appropriate tools strategically

**MP7:** Look for and make use of structure

[**MP8**](http://www.corestandards.org/Math/Practice/MP8/)**:** Look for and express regularity in repeated reasoning

After students have had time to practice in groups or teams identifying how to rotate objects and shapes through the worksheet, we will finish day two with another benchmark assessment exit slip to once again assess student understanding. This exit slip will give students a parallelogram and ask them to draw what it would look like rotation it 90, 180, 270, and 360 labeling each diagram[HSG.CO.A.5, HSG.CO.A.3]. This part of the exit slip will assess the student understanding and ability to rotate shapes in line with the two CCSS for this learning segment. Especially, important in this part of the exit slip will be for the students to understand the difference on graph paper/Cartesian plan rotation about the center of the shape, about a vertex of the shape, or about the origin[HSG.CO.A.5]. Part two of the exit slip will have students describe as best they can the difference in rotating a triangle and rotation another shape like a square or trapezoid[MP5, MP7]. I will be looking for students to understand that with a three sided (anything other than a 4 sided figure) that the rotations will not be even and will result in odd positions of orientation. The last thing on the exit slip will be another self-assessment of student understanding asking what they understand/learn and what did they feel they did not understand.

Day three will follow similar to days one and two in that they will start with a review of previous topics, spending several moments answering questions and addressing exit slip self-assessments and misconceptions. Then day three will being the discussion of translations of objects about the Cartesian plan[HSG.CO.A.3, 4, 5]. The goal will be to have student learn how to slide or glide figures to new positions without changing the orientation or shape of the figure[HSG.CO.A.5]. They will be required to show their understanding in written and pictorial forms[MP7, MP8].

**Benchmark Assessment 3:** Write the sequence of translations to transform the original figure to the new

**Benchmark Assessment 4:** Specify a sequence of transformations that will carry a given figure onto another.

[**HSG.CO.A.3**](http://www.corestandards.org/Math/Content/HSG/CO/A/3/)**:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

[**HSG.CO.A.4**](http://www.corestandards.org/Math/Content/HSG/CO/A/4/)**:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

[**HSG.CO.A.5**](http://www.corestandards.org/Math/Content/HSG/CO/A/5/)**:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**MP5:** Use appropriate tools strategically

**MP7:** Look for and make use of structure

[**MP8**](http://www.corestandards.org/Math/Practice/MP8/)**:** Look for and express regularity in repeated reasoning

The students will once again group together and work on a worksheet after a ten minute lecture and note taking on moving various objects around the plane. I anticipate that this topic will be the smoothest and least confusing to the students. However, if they do seem to be struggling with this I have prepared time to do more practice problems and to help guide them through the worksheet. If though they are able to master the material quickly then I have also prepare another worksheet to relate all three of the day’s topics. It will require the students to look at an object and decide which of the new tools they have learned will be used in each situation[MP5, MP7, MP8, HSG.CO.A.5].

To help deepen the student understanding of this learning progression, I will introduce a game for this activity were groups will compete to solve each problem I present with the document camera at the front of the room, racing to the answer and providing justification in the form of verbal feedback, requiring the proper use of the vocabulary learned in all three learning segments[MP5, MP8, HSG.CO.A.4]. Lastly, the day will end with a third benchmark assessment exit question similar to the game, requiring student to write out step by step answers to the problems instead of verbal answers they provided during the game[Benchmark 3]. Again I will include the usual self-assessment questions at the end of the exit slip.

Day four will simply be a review of all the topics, answering any misconceptions from the previous days and the day before exit slip. I will spend the first half of the period playing another game similar to day three’s with even more challenging questions, but this time the students will not be grouped so as to help prepare each student to stand on their own with the lessons[MP5, HSG.CO.A.5]. Then the last half of the period will be a summative benchmark assessment of the four days of instruction in the form of a quiz. The quiz will simply be problems from the previous days that the student must now solve and write answers to on their own[Benchmark 4, HSG.CO.A.5].

All four lessons in this learning progression connect together to help student understand the importance of reflection, rotation, translation, and lines of symmetry. The progression will introduce new topics every day as well as reinforce previous discussions. Thus, student will be continuing to build upon previous knowledge to help meet the three common core standards mentioned above as well as exercise their understanding with the mathematical practices. With each day’s benchmark assessment in the form of exit slips, I will gain a clear picture of my students and their progress towards understanding[HSG.CO.A.5, HSG.CO.A.4, HSG.CO.A.3].

**Benchmark Assessment 4:** Specify a sequence of transformations that will carry a given figure onto another.

[**HSG.CO.A.5**](http://www.corestandards.org/Math/Content/HSG/CO/A/5/)**:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**MP5:** Use appropriate tools strategically

**MP7:** Look for and make use of structure

[**MP8**](http://www.corestandards.org/Math/Practice/MP8/)**:** Look for and express regularity in repeated reasoning