High School: Algebra
Reasoning with Equations & Inequalities

This learning progression will be taught in a rural high school 10^{th} grade Algebra 1 classroom. The school is a public title 1 school. There are no scripted textbook or required curriculum at this school. However, the school does use the Common Core State Standards to guide their lessons and assessments. The school population is largely Caucasian, with the next largest ethnic group being Hispanic. There are approximately 950 students enrolled at this high school and 30.9% have free or reduced lunch. This class is averagely paced for the age and level of the students and it is a year-long class with each class lasting 50 minutes. There are 22 students in the class, 12 girls and 10 boys. The book that will be used is Algebra 1. McDougal-Littell, Larson, et al. 2004.

The Common Core State Standards that will be applied in this learning progression are HSA.REI.B.3, HSA.REI.B.4.A, and HSA.REI.B.4.B. Additionally, MP.4, MP.5, and MP.7 are the mathematical practice standards that align with this learning progression. The students have been introduced to the basics of what an equation is, how an equation differs from an expression, the vocabulary for categorizing polynomials, and the steps for solving a simple equation. They know that what you do to one side of the equation, you must also do to the other. Additionally they understand what a variable is and how to manipulate the numbers and variables in an equation to express an equation in terms of a specific variable. Through this learning progression, students will become more proficient in manipulating equations in order to solve for a variable. The students will learn what an inequality is and how they are similar and different to a standard equation. They will review how to graph linear equations and inequalities and quadratic equations and be introduced to how graphing the equations can assist in solving them. Finally, the students will be taught how to solve quadratic equations with one variable through inspection, completing the square, factoring, and the quadratic formula. The students will be given a brief introduction to complex numbers during this learning progression but that topic will be further addressed at a later time.

The students are fairly persistent in applying math to solve problems. Most of them are interested in learning but for the sole reason of simply getting and “A” in the class. There is one student in the class who is interesting in becoming a math teacher while the other students are interested in a variety of different things such as being an artist, a professional athlete, a chemist, and a chef. With enough practice, 90% of the class believes in their ability to learn mathematics. However, the first day of a new topic always leaves the majority of the students with a confused look on their face and doubt in their minds. As for believing that math is sensible, useful, and worthwhile, most of the students in this class do not see themselves using math outside of this classroom, aside from my one potential math teacher. There is a common “why do I have to learn this?” theme. They don’t typically argue about doing their work and learning, but if I am able to make a connection to the “real world” them I am better able to keep their attention.

I am a strong believer in having the students do “on your own problems” during the lecture. This is based upon Vygotsky’s Social Learning Theory. The students will have a chance to watch me perform the steps to solving a problem, and then they will have a stress-free chance to model the steps I took to solve a similar problem. I will typically then ask for volunteers to come to the front and demonstrate how they did the problem. There are no mistakes in my classroom, only learning opportunities. In order to keep the students engaged and learning, I use partner and group work. The use of collaborative learning as a teaching strategy is supported by much research. When a students who knows and understands the material is paired with someone who is struggling, formulating explanations to help their struggling partner helps to strengthen the students personal understanding (Webb, Farivar, & Mastergeorge, 2002). Alternatively, if neither of the students necessarily know the correct answer, using each other as resources to solve the problem has been shown to increase learning and elicit correct responses (Smith, Wood, Adams, Weiman, Knight, et al. 2009). I have two students with IEPs, these students require extra attention from the teacher, word processing devices for essay questions, and calculators given upon request. Additionally, I have three ELL students in this class. According to the book *Content Strategies for English Language Learners* by Jodi Reiss, "ELL's increase their opportunities for academic success when they interact with native English-speaking peers to negotiate the meaning of both language and content. Working in pairs of small groups serves to widen the language learner's zone of proximal development" (2012 p. 47). Therefore, having my ELL students work in small groups or partners for collaborative learning assists in their English language acquisition and mathematical understanding.

Reasoning With Equations & Inequalities

Solve Equations and Inequalities in One Variable

Since all of the students have come from different mathematical backgrounds, I want to start the lesson with a quick review to make sure that all of the students understand and are on the same page. In order for the students to learn how to solve equations and inequalities, they must first know what those to vocabulary words mean. According to current research, preteaching vocabulary is a highly effective strategy to teach all students, especially those with mild to moderate disabilities (Berg and Wehby, 2013). Students should be familiar with what an equation is, but they may not know what an inequality is and what the symbols look like and represent. The tip to think of the < and > signs as packmen who want to eat the bigger number always seems to help the students who are struggling to remember which symbol to use. In order to compare and contrast inequalities and equations, I will write an example of each on the board and then graph them so they can see that an equation just has one solution while an inequality has a range of solutions. Using multiple representations such as a graph and numerical representation helps the students to make conceptual connections between the ideas.

After students understand the similarities and differences between equations and inequalities, they will be introduced to the mathematical procedures involved in solving them[1]. The students will be reminded that what they do to one side of the equation, they must also do to the other. Prior to this lesson, students have learned that solving equations is a process of reasoning and they have been able to construct viable

[1] **HSA.REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

arguments for each step that they are taking. However, the equations that the students have worked with previously are very simple. Now, the students will learn the mathematical process used to solve more advanced linear equations. Through direct instruction, they will learn strategies on which term to get rid of first in order to isolate the variable. I will stand at the podium and model how to solve a few linear equations. Then, I will have the class walk me through the steps of solving another example that I provide. I will then inform the students that solving inequalities is extremely similar to solving equations, the only difference is that you must flip the inequality sign when multiplying or dividing by a negative number. I will then model how to solve a couple inequalities. Finally, before the students are assigned their activity, I will give the students two “on your own” problems to complete in their notes. One problem will be an equation and the other will be an inequality. They are able to work with their neighbor if they are stuck. I will then have a couple volunteers come to the board to teach the class how they did the problem. For my struggling and IEP students, I will write the steps to solving single variable equations and inequalities on a notecard and allow them to carry that around with them as they compete the activity.

I will be using a solving multi-step equations and inequalities station maze activity to help the students gain practice on solving linear equations and inequalities. The students will all start on one station and then be required to work in groups to solve the problem at the specific station that they are at. The problems will be multiple choice and each of the options will have another station to go to. If the students choose the correct answers, they will be able to go through each of the stations with ease. If they choose an incorrect answer, they will find themselves back at a station that they have already been at and they then be required to find their mistake. I will be walking around and informally assessing the students as they proceed from station to station. I will also be a tool that they can use if they need help. The students will be able to informally assess themselves because they will know that they did something wrong if they find themselves back at a station that they have already been at (see page 7 for station questions).

The students will be required to complete an exit task as a benchmark[2] assessment. By having one inequality and one equation, I can quickly determine level of understanding and if I am able to move on. Additionally, the word problem challenges the students and requires them to use their problem solving skills to create an equation modeling the problem and then use their procedural skills to solve it and find an answer to a real-world problem.

[2] **Benchmark:**

Solve for x

1. $3x + 2 = \frac{x}{4}$
2. $-2(x + 3) < 10$
3. Marcia has just opened a new computer store. She makes $27 on every computer she sells and her monthly expenses are $10,000. What is the minimum number of computers she needs to sell in a month to make a profit? Express this problem as an equation and then solve for the minimum number of computers that needs to be sold in a month in order to make a profit.

After the students understand how to solve linear equations and inequalities, they will be introduced to solving quadratic equations with one variable. At first, we will review what a quadratic equation looks like numerically and graphically and how that compares to a linear equation. I will point out that the place in which the graph crosses the x-axis is the "root" of the equation, which is what we're solving for. Students will be able to connect this idea of finding where graphs cross the x-axis to when we solved linear equations. After students have been reminded of the properties of quadratic equations, I will begin to model how to solve quadratic equations by completing the square and by inspection[3]. I do not expect the by inspection part to take very long because the students are simply isolating the x^2 term and then taking the square root of both sides. To support students learning, I will use pictures to demonstrate what completing the square actually means. Having the students draw these pictures will give them a deeper understanding of what the procedural steps are achieving. Having the students draw a picture, write the equation symbolically, and also orally discuss the problem with their partner allows the students to successfully model with mathematics[4]. The process of drawing the picture and writing the equation symbolically is shown in the worksheet on page 8. Additionally, I will write down the procedural steps that the students will have to take to complete the square. Just like we did with the solving linear equations and inequalities, I will first model the steps for the students, then have them walk me through an example, and finally provide them with "on your own" problems to do in their notes in which I will have volunteers come to the board and explain to the class how they got their answer. A common mistake is for students to attempt to start completing the square before they have made sure that the constant multiplier on the squared term is a 1. In order to remedy that problem, I will write the steps on the board for the students to follow. The first step will always be to assure that there is only a 1 multiplied by the x^2 term.

In order for the students to really grasp the concept of completing the square, they need to have an opportunity to practice the steps and commit these steps to their long-term memory. That is why I will provide the students with a scaffolded worksheet as a formative assessment for this standard. On the first line there will be the equation, then the students will be required to draw a picture. This picture will show them visually what completing the square means. I will have modeled how to draw this picture during the lecture. Finally, they will be required to write the number that they must add to both sides, do so, and then write the equation in the $(x - p)^2 = q$ form. By doing this, they are demonstrating their ability to attend to precision[5]. For example, if a student wrote $(x - p^2) = q$, then they are not accurately attending to precision. Once they have done that, they will solve the problem in

[3] **HSA.REI.B.4.A** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

[4] **CCSS.MATH.PRACTICE MP4** Model with Mathematics

[5] **CCSS.MATH.PRACTICE. MP6** Attend to precision

order to find the roots. Roots must be written in "x=" form, this is another way in which they demonstrate their ability to reach standard MP4. For example, if the students simply wrote 4 and 7 as their answer, then they're unable to attend to precision. In order to demonstrate their ability to reach this standard, they will have to write " $x = 7\ and\ x = 4$". The worksheet can be found on page 8.

After the students have had time to practice the steps to completing the square on their worksheet, they will be provided with an exit slip[6] that I will use as evidence of their understanding. By having the students solve a linear equation and a quadratic equation, without mention of the method in which to solve, I am assessing the students on their ability to differentiate and execute solving methods. The students will have to use their problem solving skills and knowledge about quadratic and linear equations to decide if they should simply isolate x or if they must compete the square. Based on their ability to answer the exit slip and correctly complete the worksheet, I will know if I need to review completing the square or if I can move on to factoring.

After the students have mastered completing the square, they will learn another tool for solving quadratic equations: the quadratic formula[7]. At the beginning, in order to build conceptual knowledge, I will show the students how to derive the quadratic formula from the process of completing the square. However, I will not require them to personally know how to derive it, I just want them to know that it can be done. I will emphasis that the quadratic formula works for any quadratic, and unlike completing the square, it doesn't get too terribly nasty looking with fractions. To help with procedural fluency, I will go through a few example where we just look at the quadratic equation and recognize "a", "b", and "c". Then I will model how to correctly put the numbers into the formula. We will chant the song in class so the students have a chance to repeat the formula over and over again so they will hopefully commit it to their long term memory. I will play a quadratic formula song[8] to help the auditory learners to remember the quadratic formula and also to provide a comedic relief to the day. I will provide my struggling students with a notecard that they can tape to the top of their desk with the quadratic formula written on it. The exit slip[9] will ask the students to identify "a", "b", and "c" and then use both completing the square and the quadratic formula to assess their ability to do both. It will also show to them that it doesn't matter which method they use, both will give them the same answer.

The final topic that the students will learn in this learning progression is how to use factoring to solve quadratic equations[10]. This topic as a whole could almost be its own learning progression; however, for this progression, I will only teach them how to factor equations when the

[6] **Exit Slip:**
Solve the following equations:

1. $6k - 18 = 6(1 + 3k)$
2. $x^2 - 6x - 3 = 0$

[7] **HSA.REI.B.4.B** Solve quadratic equations by…the quadratic formula…(partial standard).

[8] **Quadratic Formula Song** https://www.youtube.com/watch?v=z6hCu0EPs-o.

[9] **Exit Slip**
Identify "a", "b", and "c" and then solve by completing the square and the quadratic formula. Show your work

$$3x^2 + 10x - 3 = 0$$

[10] **HSA.REI.B.4.B** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

coefficient of the squared term is 1. The students have been taught how to use FOIL to distribute and multiply binomials. I will use this previous knowledge in order to build upon the idea of factoring. Basically, we are just undoing what we did when we distributed. Again, the students will be labeling the coefficients of the terms as “a”, “b”, and “c”. Therefore, they can use what they learned in the last lesson to guide them on correctly labeling the terms. The fact that the students have to multiply a and c and then find two factors of that product that add together to equal b allows the students to practice meeting the standard MP7[11] . For example, one the Bingo card problems would be factor $z^2 - 4z + 12$. Therefore, students would have to make use of the structure of positive and negative numbers[11]. They will have to notice the pattern that if the constant term is negative, as in the problem above, then they will have (z - some number) times (z + some number). Additionally, they will have to notice the pattern that, if the z term is negative then the largest factor will be negative and if the z term is positive then the largest factor will be negative. I will model the steps to factoring expressions and will provide them with opportunities to practice these steps during the lecture in their notes and on the board. Then, I will show them how they can use that skill to solve factorable equations. I will teach them the vocab word discriminant and how they can use the discriminant to determine whether or not an equation is factorable. Using the discriminant will provide struggling students with additional connections to previously learned material: the quadratic formula.

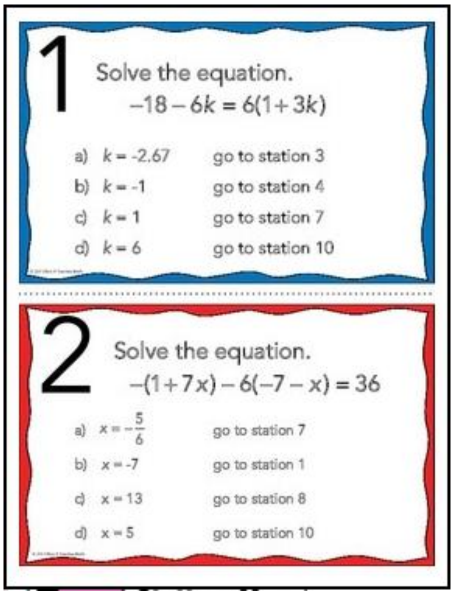
The formative assessment that I will use for this standard is a factoring Bingo game. The students will be in pairs and each will have bingo sheets in front of them and each square will have an equation in factored form. I will supply the students with an unfactored equation and then they will have to factor it and then see if they have the answer on their sheet. If they do, they can mark it out. Additionally, to review, I will provide the students with already factored equations and they will have to distribute the binomials and see if they have the unfactored quadratic on their sheet. Using a game such as this allows my students to be engaged and allows my ELL students to engage in cooperative learning which can both enhance their English skills and math skills. As the students are working, they will have white boards in which they will be required to show their work. By walking around looking at these white boards, I will be able to assess whether the students understand how to factor equations when the leading coefficient is 1. The factoring Bingo activity is on page 9.

[11] **MATH.PRACTICE.MP7** Look for and make use of structure

Worksheets and Activity Materials

Solving Multi-Step Equations Activity

Disclaimer: This activity was a free download from "Mrs E Teaches Math" on teacherspayteachers.com. I would change it so 5 of them were represented as inequalities instead of all equations.

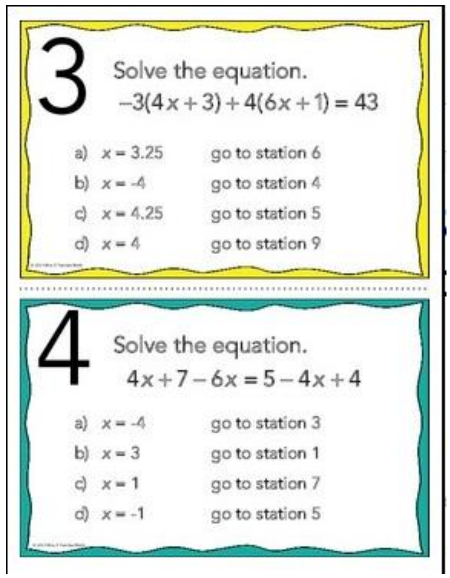


1 Solve the equation.
$-18-6k=6(1+3k)$

a) $k=-2.67$ go to station 3
b) $k=-1$ go to station 4
c) $k=1$ go to station 7
d) $k=6$ go to station 10

2 Solve the equation.
$-(1+7x)-6(-7-x)=36$

a) $x=-\frac{5}{6}$ go to station 7
b) $x=-7$ go to station 1
c) $x=13$ go to station 8
d) $x=5$ go to station 10

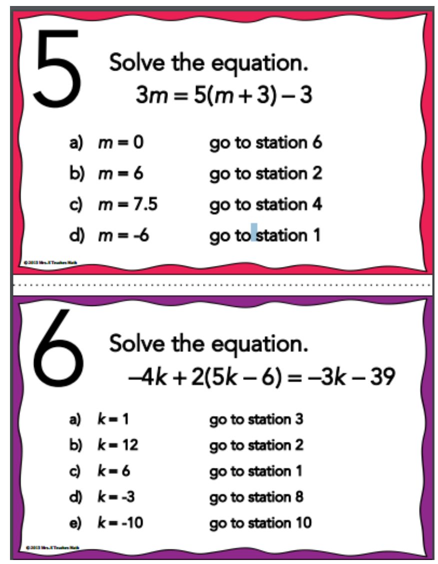


3 Solve the equation.
$-3(4x+3)+4(6x+1)=43$

a) $x=3.25$ go to station 6
b) $x=-4$ go to station 4
c) $x=4.25$ go to station 5
d) $x=4$ go to station 9

4 Solve the equation.
$4x+7-6x=5-4x+4$

a) $x=-4$ go to station 3
b) $x=3$ go to station 1
c) $x=1$ go to station 7
d) $x=-1$ go to station 5

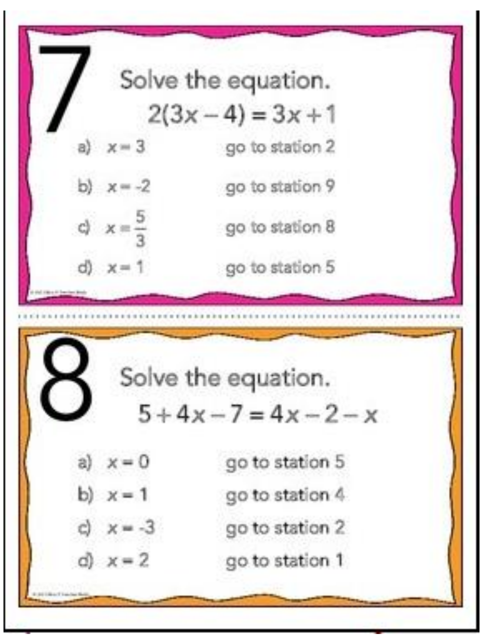


5 Solve the equation.
$3m=5(m+3)-3$

a) $m=0$ go to station 6
b) $m=6$ go to station 2
c) $m=7.5$ go to station 4
d) $m=-6$ go to station 1

6 Solve the equation.
$-4k+2(5k-6)=-3k-39$

a) $k=1$ go to station 3
b) $k=12$ go to station 2
c) $k=6$ go to station 1
d) $k=-3$ go to station 8
e) $k=-10$ go to station 10

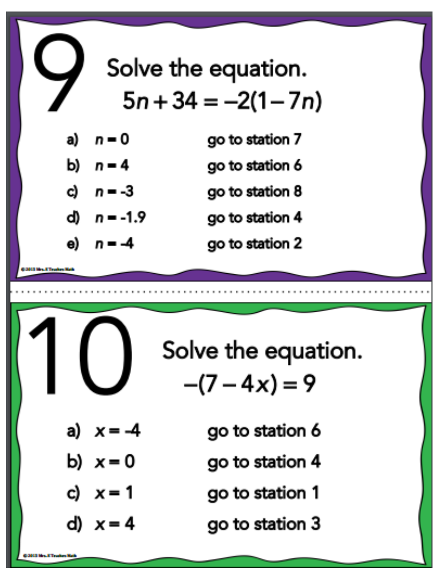


7 Solve the equation.
$2(3x-4)=3x+1$

a) $x=3$ go to station 2
b) $x=-2$ go to station 9
c) $x=\frac{5}{3}$ go to station 8
d) $x=1$ go to station 5

8 Solve the equation.
$5+4x-7=4x-2-x$

a) $x=0$ go to station 5
b) $x=1$ go to station 4
c) $x=-3$ go to station 2
d) $x=2$ go to station 1



9 Solve the equation.
$5n+34=-2(1-7n)$

a) $n=0$ go to station 7
b) $n=4$ go to station 6
c) $n=-3$ go to station 8
d) $n=-1.9$ go to station 4
e) $n=-4$ go to station 2

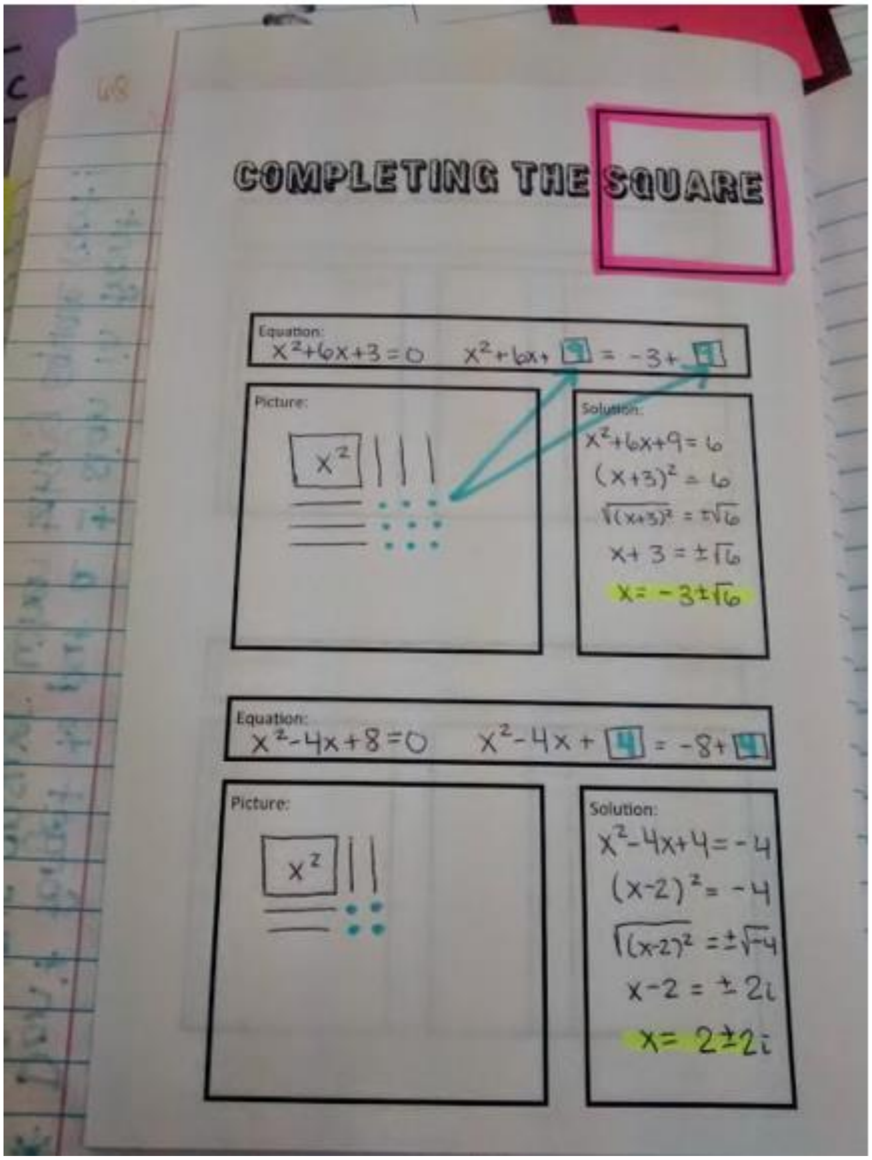
10 Solve the equation.
$-(7-4x)=9$

a) $x=-4$ go to station 6
b) $x=0$ go to station 4
c) $x=1$ go to station 1
d) $x=4$ go to station 3

Completing the Square worksheet:

Taken from http://mathequalslove.blogspot.com.au/2015/05/algebra-2-solving-quadratics-inb-pages.html

The equations will be written on the board and the students will copy them from the board onto their equation box. They will then complete the picture and solution. They will have 4 pages scaffolded like the worksheet shown below.



Factoring Bingo Game

An example of a student's card would be:

$(x-3)(x-1)$	$(x+9)(x+7)$	$(y-2)(y+4)$	$(2j)(j+15)$	$30x(x-1)$
$(x+5)(x-4)$	$(z+2)(z-6)$	$(k-11)(K+8)$	$-10(x+9)$	$(y+29)(y+4)$
$x^2+2xy-15y^2$	$(x+3)(x-2)$	FREE	$(x-7)(x-5)$	$(t+19)(t-3)$
$3x-9$	$(y-4)(y-1)$	$(x+9)(x+2)$	$2(x+y)$	$5x^2+10x$
$6x^2+13x+6$	$4x^2-11x-3$	$(x+5)(x-4)$	$x(x-2)$	$(y-17)(y-9)$

And I would pull problems out of a hat and display them on the overhead
so all the students can see. First person to get a Bingo gets extra credit.

Works Cited

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