**High School: Geometry**

*Special Triangles*

#### This learning progression was designed primarily for 9th grade students in an honors geometry course. The Common Core State Standards that it will be satisfying are the following: [HSG.SRT.B.4](http://www.corestandards.org/Math/Content/HSG/SRT/B/4/), [HSG.SRT.B.5](http://www.corestandards.org/Math/Content/HSG/SRT/B/5/), [HSG.SRT.C.6](http://www.corestandards.org/Math/Content/HSG/SRT/C/6/). While the math practices that we will be implementing are: [MP1](http://www.corestandards.org/Math/Practice/MP1/), [MP2](http://www.corestandards.org/Math/Practice/MP2/), and [MP8](http://www.corestandards.org/Math/Practice/MP8/).

Common Core State Standards

**Content Standards**

**Prove theorems involving similarity**

CCSS.MATH.CONTENT.HSG.SRT.B.4

Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

CCSS.MATH.CONTENT.HSG.SRT.B.5

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Define trigonometric ratios and solve problems involving right triangles**

CCSS.MATH.CONTENT.HSG.SRT.C.6

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

**Mathematical Practices**

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

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 Figure 1

#### The curriculum these students are going through is produced by Holt. More specifically, the students are working from the 2009 Geometry textbook from the same publisher. They have already conquered identifying triangles by their sides and by their angles. The students have also already learned how to determine if various triangles are similar or congruent to each other. Because these skills were conquered only a matter of weeks ago, I will be asking my students review questions throughout the learning progression on those topics. Meanwhile, the central focus of the lesson progression presented here will be understanding 30, 60, 90 and 45, 45, 90 triangles as well as finding consistent ratios that appear in these triangles. The students will then discover how to use these ratios to find the two missing side lengths when given one side length and the angle measures of a triangle.

#### At the very beginning of the lesson progression the students will be given diagram 1, which can be seen in the left margin, and guided through deriving the common ratios found in 30, 60, 90

#### Triangles using the Socratic method. To ensure that the Common Core State Standards HSG.SRT.B.4 and HSG.SRT.B.5 are met and to help the students review the material they recently conquered, I will begin by asking them, “What congruence theorem can we use to prove that the two halves of the equilateral are congruent?” This will function as the beginning of the 30, 60, 90 triangle theorem proof. The lesson will then continue and I will ask the students to find the length of the long leg, short leg, and hypotenuse based once again on diagram 1. Since we will derive the ratios using a general case, students will have to implement mathematical practice 2 and reason abstractly in order to solve the problem. Then after this abstract reasoning is completed students will need put mathematical practice 1 into place when they attempt to make sense of what the abstract symbols they just derived mean, how they relate to one another, and how one can use them to determine unknown quantities given a single side length.

#### Organizing the information, they just derived into useful, understandable and therefore usable relationships, rather than completely abstract

####  symbols that mean nothing to the students, will likely take quite a bit of guidance. To help students with this process I plan to enter into a short discourse with the class about what the symbols represent and why the relationship between these quantities will hold true for any 30, 60, 90 triangle. After this I create a learning environment where the students are able to assimilate the information they just learned by actively implementing it. The students will all receive white boards and they will be told to draw a 30, 60, 90 triangle on their board. I will then give them the length of the short side of the triangle. As seen in figure 2, this value will be equivalent to *X.* The students will then simply have to plug this *X* value into the expressions for the long leg and hypotenuse to determine how long all of the legs are. They will then label the side lengths of the triangle they drew according to their calculations. When they finish the problem they will hold up their whiteboard so that I can formatively assess how each individual is doing and decide if further explanation is needed before moving onto more difficult problems. During this process I should be able to pick up on any misconceptions that may arise quickly so that I can address them before they become prevalent within the class.

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 Figure 2

Benchmark Assessment 1:

If the short leg is 5 units long find the length of the hypotenuse and long leg.

Benchmark Assessment 2:

If the hypotenuse is 15 units long find the length of the short and the long leg.

Benchmark Assessment 3:

If the long leg is $2\sqrt{3}$ find the length of the short leg and the hypotenuse.

Benchmark Assessment 4:

If the long leg is 21 units long find the length of the short leg and the hypotenuse.

#### After students understand what to do when given the short leg I will move on and tell them the length of the Hypotenuse. They will have to use their own reasoning abilities to discover that they first need to solve for X by dividing their value by two. Then finding the long leg will be relatively easy for them. The class will once again use the whiteboards to demonstrate their understanding or lack thereof so I can make informed decisions about when to progress to the next step in the lesson. When I observe that the students are proficient at finding the side lengths when given the hypotenuse I will proceed to giving them the long leg. At first when I give them the long leg the measure will always include the radicle of 3. This will make isolating X far easier for them. When they begin to consistently solve for X in these problems I will give them a long leg length that is a whole number. They will have to use math practices 1 and 2 in order to come to the conclusion that they must first to set their given number equal to $X\sqrt{3}$ and solve for X. Doing this will allow them to solve for the short leg and hypotenuse. Once the students accomplish and successfully complete another problem of the same type I will ask them to brainstorm with a partner and try to see if they can figure out a shortcut to completing problems of this type. This procedure will force them to use math practice number 8. After the students realize all one must do to solve for X is take the given number and multiply it by $\frac{\sqrt{3}}{3}$ I will give them a few more problems of this type until they are completing them rapidly and error free. Once the students meet this benchmark we will move on to the next portion of the lesson progression.

#### After conquering 30, 60, 90 triangles the students will be able to move onto working with 45, 45, 90 triangles. Just as before students will first derive the common ratios found in 45, 45, 90 triangles using figure 3 which can be found in the left margin. I will lead them through the process using the Socratic method. During this process I will make sure we discuss what congruence theorems can be used to prove that the two halves of the square are congruent. This will be the beginning of the 45, 45, 90 triangle theorem proof and will ensure that HSG.SRT.B.4 and HSG.SRT.B.5 are met.

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 Figure 3

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 Figure 4

Benchmark Assessment 5:

If the legs are each 11 units long, how long is the hypotenuse?

Benchmark Assessment 6:

If the hypotenuse is $5\sqrt{2}$ units long how long are the legs?

Benchmark assessment 7:

If the hypotenuse is 13 units long, how long are the legs?

#### The students will arrive at figure 4 and from there I will begin by giving them lengths for the legs of the triangle. Just as they did in the previous activity the student will write the length of the remaining side or sides on their whiteboard and hold it up for me to check their understanding. Once they pass benchmark number 5 and therefore are proficient at this I will give them the length of the hypotenuse. The measure of this length will include a radical of 2 in it to simplify the problem for the students. Once they seem to be understanding I will check their understanding using benchmark number 6. Then I will precede to give them a whole number for the hypotenuse and have them, like before, solve for the legs and then come up with a shortcut so that they don’t have to set their given number to $X\sqrt{2}$ and solve for X each time. After a few more practice problems I will give them benchmark assessment number 7. This will mark the completion of this particular lesson progression.

#### At this point students will have just begun to work towards completing HSG.SRT.C.6 and they will be set up to begin working with trigonometric functions. This will be the topic of the next lesion progression that is completed.