**High School: Calculus**

***Modeling a Population Model***

This learning progression will be applied to a high school calculus classroom, normally for 12th graders, that uses Khan Academy as a reference for the curriculum. The following Common Core State Standards aligned with this progression are HSF.LE.A.1.c and HSF.LE.A.4. The standards for mathematical practice that will be used are MP1: Make sense of problems and preserve in solving them, MP4: Model with mathematics, and MP6: Attend to precision.

The students have had experience with finding the derivative and integral of a function. They have been introduced to the rules of integrating a function that produces a natural logarithmic equation, for example, . This learning progression will build on this knowledge by applying this specific type of function to modelling a population. This will build an understanding of the real-world applications of logarithmic functions and where they come from. The students will be working cooperatively in groups on example problems to further their own ability of the procedural steps of integrating. This lets the students communicate with their peers so that they can continue to increase their knowledge with discussions about each of the practice problems. Although most of the progress requires the students to work cooperatively, the final portion will be assessed on how well each individual student understands the concept and procedures. This is displayed in a final activity where each student must model a population by integrating by parts of a function.

**Part 1: Reviewing Integrating Functions.**

The first part of the learning progression is a review portion so that the students can determine their own skill level of integrating functions, as well as for me to determine the speed of the progression. This is important for me to use this time to challenge the students on their procedural steps so that they can progress further into the lesson. The practice problems will be provided on a worksheet that build on each other by starting off with simple integrations to building into problems that require more advanced methods, such as substitution, to integrate a function. Students who are struggling with the problems will be able to get assistance from their peers or individual help from me so that they can gain mastery of the procedural steps of integration. In order to move on to the next section of the progression, there must be a ninety percent of procedural fluency shown by the students from their integrations on the worksheet as a formative assessment.

**Part 2: Integration by Parts and Modeling Example.**

This portion of the learning progression will introduce the students to the idea of separating a function by parts. This will begin with using Khan Academy as a source of information of integration by parts. This will give the students a basis of understanding the concepts and procedures of integration by parts. After the students have watched the selected videos, I will go over example problems with the whole class. Such problems will begin simple, for example:

This will be a good example to start off with so that the students are able to see the simplicity of the process in order to get certain variables to a specific side. The example also shows the process of a simple problem that requires integration by parts to create a function. From this example, the students are given two problems written on the white board that they must integrate by parts to find a differential equation for *y* as a function of *x*. These example problems will be and so that the students can practice with simpler integrations on the first problem and a natural logarithmic solution on the second problem. The students will take lead on these problems by working with each other in groups. These practice problems will allow for the students to address the Standard for Mathematical Practice of making sense of problems and persevere in solving them because they are able communicate their mathematical process with their peers of integration by parts of the two practice problems.

[CCSS.Math.Practice.MP1](http://www.corestandards.org/Math/Practice/MP1/) **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

After the students complete these two exercise problems, the idea of one-dimensional models will be introduced relating to integration by parts with a general differential population equation. For example if we have a rate *r*,

[CCSS.Math.Content.HSF.LE.A.4](http://www.corestandards.org/Math/Content/HSF/LE/A/4/)

For exponential models, express as a logarithm the solution to *abct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology

This will introduce a general solution to a one-dimensional model that only changes positively. The example displays some misconceptions and ways to simplify the equation by setting . This is an important change that simplifies the equation along with showing the students that is just a constant regardless if it is positive or negative, thus we can change the value to . By finding this general solution, this aligns directly with the CCSS HSF.LE.A.4 by evaluating an exponential function with a rate of change of a time period. As a way to show that this value displays the initial value of the population, we can assume at the beginning of the time period such that .

From this example, the lesson can scaffold to include a more realistic population where there is a carrying capacity that is applied to a population model. The students will be able to adjust the general differential equation to include a capacity for the population. This will lead to the discovery of where *r* is the rate of change and *K* is the carrying capacity. After practicing with population models and seeing what happens to the graphs that have different rate of changes with a set carry capacity, this part of the learning progression will conclude with an exit slip. This formative assessment of an exit slip will include hinge questions about each students understanding of the concepts and procedures of integration by parts and population models and the CCSS of HSF.LE.A.1.c by having the students recognize the growth of the model. If there is a ninety percent of accuracy from this exit slip, then the class will move on to the population model activity. This exit slip allows for each student’s understanding to be addressed so that there is a possibility to come back and talk about each part where a misunderstanding occurs.

[CCSS.Math.Content.HSF.LE.A.1.c](http://www.corestandards.org/Math/Content/HSF/LE/A/1/c/)

Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**Part 3: Population Model Activity**

[CCSS.Math.Practice.MP4](http://www.corestandards.org/Math/Practice/MP4/) **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life.

[CCSS.Math.Practice.MP6](http://www.corestandards.org/Math/Practice/MP6/) **Attend to precision.**

Mathematically proficient students try to communicate precisely to others.

The students will be given a problem where they must model a population of the world that requires them to use a specific carrying capacity. They will be given an initial population along with different values of rate of change. This will assess each student’s ability to perform the procedural steps of integration by parts and applying the different rates of change. They will also be assessed on how well they can conceptualize what different rates of change will do to each differential equation graphically. This will address the Standard for Mathematical Practice, Attend Precision, by having the students communicate their capability of understanding the procedures of integrating by parts and the concepts that are related to modeling a population. It also addresses the Standard for Mathematical Practice of Modeling with Mathematics by having the students take a population and model it with a differential equation. The goal level of accuracy of this activity is eighty percent as this is the summative assessment of the learning progression. The students will be assessed on their ability of achieving the CCSS that align with this progression.