Vocational Geometry
Solve Real Life and Mathematical Problems Involving Area, Surface Area, and Volume

This learning progression will be taught in an applied geometry classroom at Ellensburg High School. The school is a public title 1 school. There are no scripted textbook or required curriculum at this school. However, the school does use the Common Core State Standards to guide their lessons and assessments. The school population is largely Caucasian, with the next largest ethnic group being Hispanic. There are approximately 950 students enrolled at this high school and 30.9% have free or reduced lunch. This class is slowly paced in comparison to other geometry classes and it is a year-long class with each class lasting 50 minutes. The students are only receiving a general math credit for this class, they are not receiving a geometry credit. There are 28 students in the class. The class will be using the *Online CORD Geometry Learning in Context Fourth Edition* book.

The Common Core State Standards that will be applied in this learning progression are 7.G.B.4, 7.G.B.6, and 8.G.C.9. Additionally, the mathematical practice standards that align with this learning progression are MP4; model with mathematics, MP6; attend to precision, MP7; look for and make use of structure. Since this is a vocational math class in which the students are not receiving geometry credit, the students' prior content knowledge is varied. The decision to use the above mentioned standards is based upon the fact that the students struggle to meet the standards commonly used for their age and grade level. Therefore, we have to choose standards that are attainable, even if they are 7th and 8th grade standards. Most students know how to graph numbers on a coordinate plane, how to identify different shapes such as spheres, cylinders, and cones, and how to determine slope of a line. Some of the students may already know that pi is the circumference of a circle divided by the diameter, while other students may struggle with even identifying what the circumference of a circle is. Through this learning progression, students will gain a more conceptual understanding of where the irrational number pi comes from and how to use that number to find area, surface area, and volume. Additionally, students will learn how to find area, surface area, and volume of objects not involving pi. They will have to use their problem solving skills to apply these formulas to real-world figures, including some of their own art work.

The students are not persistent in applying math to solve problems. They are in this class because they have struggled with getting math credit from other classes and they frequently get discouraged by their perceived lack of mathematical knowledge. The students know that they are in a low level math class; therefore, their confidence in their ability to learn mathematics is poor. It is often difficult to get the students to participate in lessons. As for believing that math is sensible, useful, and worthwhile, most of the students in this class do not see themselves using math outside of this classroom. They are only interested in learning content related to their future careers, and that link must be very explicit or it may be difficult to get the students to become engaged. It has been noticed that the students are more persistent in using math to solve problems when they are allowed to work in partners or small groups; therefore most of the lessons are cooperative based.

The first lesson in this progression is a discovery based cooperative learning activity in which the students discover pi through measuring circumferences and diameters of different objects and finding the quotient. The students will then move on to learning about area and surface area of different objects. Finally, the students will use Google Earth and the formulas that they have learned to find area and volume of different objects throughout the world. The use of collaborative learning as a teaching strategy is supported by much research. When a student who knows and understands the material is paired with someone who is struggling, formulating explanations to help their struggling partner helps to strengthen the students personal understanding (Webb, Farivar, & Mastergeorge, 2002). Alternatively, if neither of the students necessarily know the correct answer, using each other as resources to solve the problem has been shown to increase learning and elicit correct responses (Smith, Wood, Adams, Weiman, Knight, et al. 2009).I have two students with IEPs, these students require extra attention from the teacher, word processing devices for essay questions, and calculators given upon request. Additionally, I have three ELL students in this class. According to the book *Content Strategies for English Language Learners* by Jodi Reiss, "ELL's increase their opportunities for academic success when they interact with native English-speaking peers to negotiate the meaning of both language and content. Working in pairs of small groups serves to widen the language learner's zone of proximal development" (2012 p. 47). Therefore, having my ELL students work in small groups or partners for collaborative learning assists in their English language acquisition and mathematical understanding.

Geometry

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To address the first standard, 7.G.B.4, students will be working on a worksheet to discover the irrational number pi. The students will go through an activity that will have them taking the circumference and diameter of various objects. After the students find the circumference and diameter of the objects, they will find the ratio between the two. The goal is for students to see that the ratios are roughly the same: 3.14. The student will also discover pi through graphing. They will use their table they created to graph circumference vs. diameter to show that the slope of this graph is pi. While this lesson is going on, the students will be asked to individually demonstrate how to find the circumference and the diameter to show they understand what those values would be. This lesson will require the students to look for and make use of structure, MP7. The students will be completing this ,mathematical practice by looking for the pattern of the ratios being the same. They will also notice that the ratios they found are equivalent to the slope of their graph they created using the circumference and diameter. Once the students notice the matter, they will be able

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MP7 Look for and make use of structure

to use the table and graph to derive an equation for circumference.

To assess the student's on this standard they will be completing and turning in the worksheet that they worked on. This worksheet is comprised of conceptual questions along with the table and graph. The students will use this to demonstrate their understanding of circumference, diameter, and pi. Another way they will be assessed is by student-teacher interactions during the lesson. The students will be asked questions such as "what is the slope of the line?" and "what pattern do you see in the ratios?"

After discovering the irrational number pi, the students will build on this information and apply it to their knowledge of area and surface area. The students have been introduced to the concept before and have an understanding of the different types of equations for calculating the area of two-dimensional shapes. This lesson is building on this knowledge and addressing the standard 7.G.B.6 by having the students moving on to finding the surface area of three-dimensional figures.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

The lesson will start with the students reviewing what they know about area and general equations with an informal discussion. This will allow for the students to get a review of some of the important points when calculating the area of two-dimensional shape. The opportunity of relating the area of two-dimensional shapes to the surface area of three-dimensional figures is presented by showing the students how to break down a prism into smaller figures to calculate the surface area. The students will be able to see this by finding that the surface area of a cylinder is the same as adding up the two circles that make up the ends and a rectangle that has the dimensions of the height and the circumference of the circles. The students will also be able to use origami prisms that they created to calculate the surface area. These models address the mathematical practice standard of MP6 by having the students use models to calculate the surface area of their figures. The students will be able to work with a live model and measure out the dimensions, which may vary from their peers in the case of using the origami prism, and calculate the surface area. Students are able to find a real-world connection

MP6 Attend to precision

with surface area.

Once the students have mastered finding area and surface area of different objects and shapes, they will be taught how to find the volume of spheres, pyramids, cylinders, cones, and cubes and how that applies to solving real-world and mathematical problems, 8.G.C.9. Some of the vocabulary for this lesson may be new to the students. They would have learned the vocabulary words pi, area, diameter, radius, base, height, and surface area previously in the learning progression. However, the terms volume, sphere, and pyramid may be new. For that reason, I will pre-teach the vocabulary in order to support the students who may not completely understand the terms and those students with disabilities. According to current research, preteaching vocabulary is a highly effective strategy to teach all students, especially those with mild to moderate disabilities (Berg & Wehby 2013). I will verbally ask the students questions to guide them towards conceptual understanding of volume. I will model a couple examples of calculating volume of different objects prior to setting the students loose to complete their activity. Calculating surface area should be review for them from the previous lesson in the learning progression. Additionally, I will write the formulas for the volume and surface area for the different objects targeted in the lesson on the board for the students to reference as needed.

The central focus of this lesson is to provide the students with a real world application of the concepts that they have been learning in this progression. This class is primarily focused on vocational skills and the students are interested in how they can apply math outside of the classroom.Therefore, this lesson highly aligns with the MP4 model with mathematics practice standard. Many of these students may go into mechanics, construction, landscaping, and welding. Therefore, being able to find the area and volume of 3-D objects is important for their future careers and everyday life. For their level of understanding of math, being able to complete this activity demonstrates their ability to model with mathematics. During this lesson, the students will be doing an activity on Google Earth accompanied by a worksheet that I created, which will be used as their benchmark assessment. In order to access the activity, they will have to download the kmz file from realworldmath.org. The program will take them around the world to 13 different locations. The students will be able to view the focus buildings in 3-D while in “street view” so they can explicitly see the real-world connections. The program will then

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

http://www.realworldmath.org/concept-lessons.html

The Leaning Tower of Pisa is a cylinderical building.

Find the volume and surface area of a cylinder given the following dimensions:

- **Height: 56 meters**
- **Diameter: 15.5 meters**

provide them with the different dimensions necessary to calculate the surface area and volume of each of these buildings. This activity has a huge cultural connection because it requires the students to tour the world. They are required to virtually visit different countries,such as France, to find the surface area and volumes of the buildings in those countries. I will strongly encourage the students to do more research on the buildings that were touched on during class.

Based on the students' distaste for math, lessons have to be fun and engaging. That is why I chose to use technology in order to guide the lesson and help assess their understanding. The margin shows an example of one of the buildings that the students will be working with during the lesson. To challenge the students and to make further cultural connections, I ask the students on the worksheet to use Google to search for a building not already on the worksheet that is cylindrical, spherical, or cone shaped. They will then have to use their problem solving skills to determine which dimensions of the building that they need in order to calculate the volume and surface area, research those dimensions, and then chose the correct formula to use.

Discovering Pi

Name:_____

Objective: Today you will discover a ratio between a circle's diameter and circumference.

Procedures:

1. Measure the diameter of each round object you are given
2. Use your string to measure the circumference of the circle. Mark the place where the string meets and measure the distance with a ruler.
3. Fill in the chart below.
4. Use the Diameter as the x-axis and Circumference as the y-axis to plot the numbers you found.
5. Find the slope of the graph you created.
6. Using the chart, calculate the ratio circumference to diameter and record it in the chart.
7. Calculate the average ratio for your circles.

Table and graph on next pages

Questions:

1. What is the average ratio of circumference to diameter?

2. Why is it important to measure circles of different sizes?

3. What observations did you make?

4. Can you derive the formula for circumference of a circle by knowing Pi is the ratio of diameter to circumference?

Table:

Circle Number	Radius	Diameter	Circumference	Ratio (Circumference/Diameter
1				
2				
3				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
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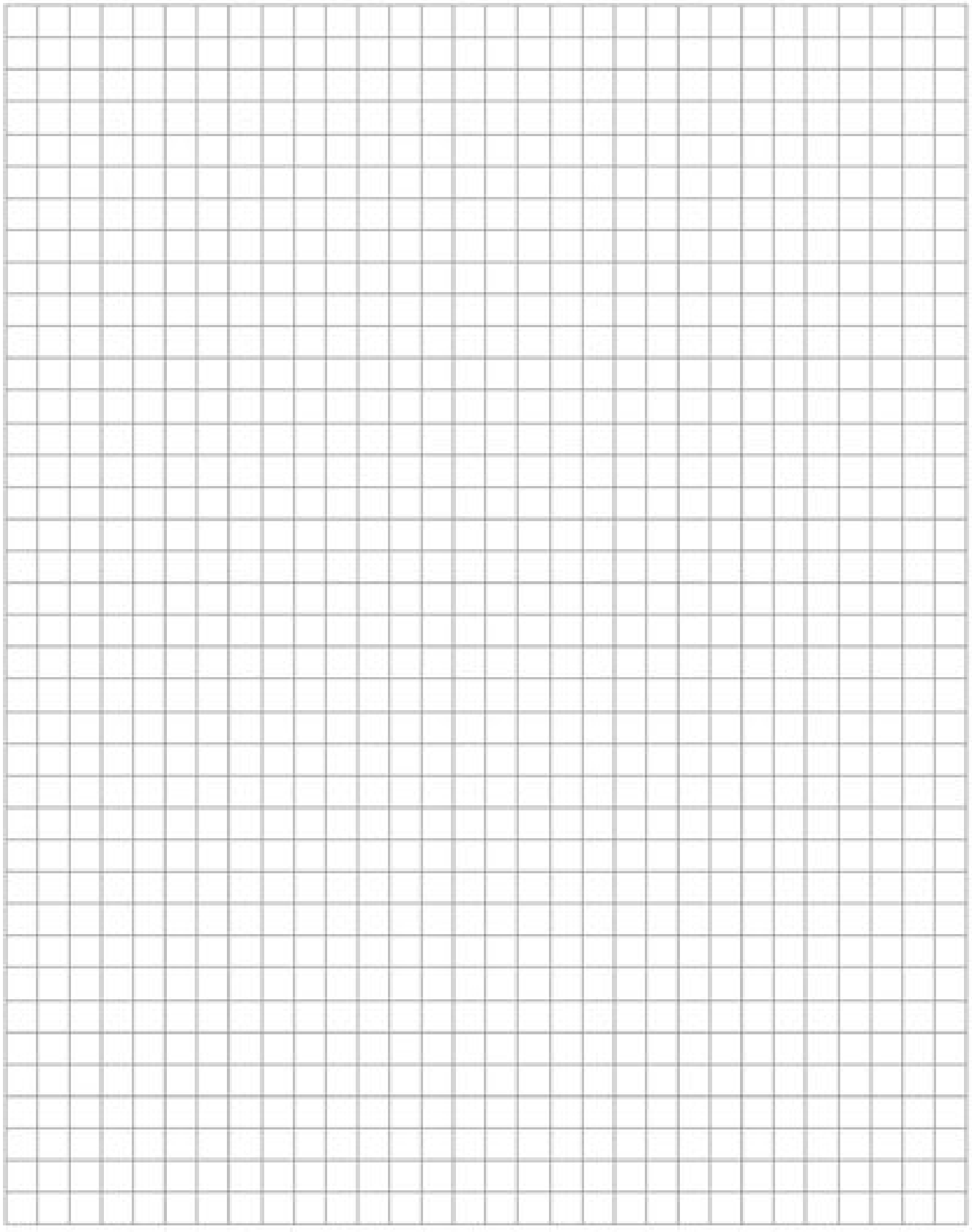
21				
22				
23				
24				

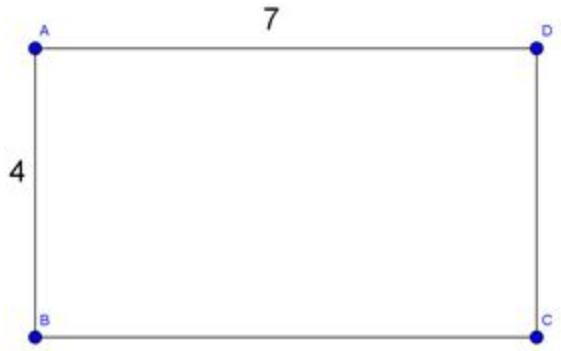
On the graphing paper, graph the circumference vs. the diameter. (circumference on the *y-axis* and diameter on the *x-axis*)



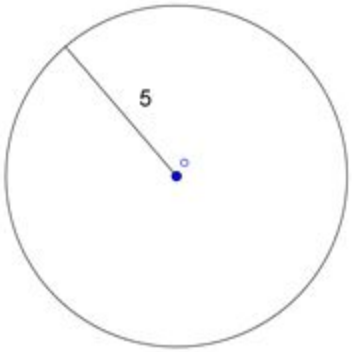
Area and Surface Area Worksheet

For problems 1-3, write the area equation for each the polygons and then find the area.

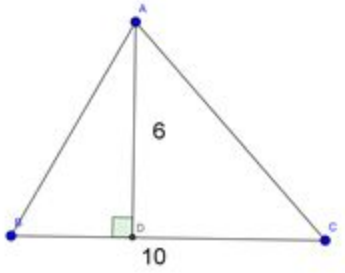
1.



2.



3.



4. Find the surface area of your origami figure. (Show all work to receive full credit)

5. What do you notice about the shapes of a cylinder for calculating the surface area?

6. Find the surface area of the given cylinder. (Hint: you will need to use a ruler to find the dimensions)

7. What is the difference between area and surface area?

GeoMath

Directions: Sign onto your google classroom page and click on the link for today's lesson. Follow the prompts given to you by GoogleEarth and record your work and answers here. Please remember, no work means no credit. Once you are finished, feel free to virtually explore the world.

1. Globe

Surface Area:

Volume:

2. Head Building

Surface Area:

Volume:

3. Transamerica Pyramid

Volume:

4. World Trade Center site

Volume:

5. Reunion Tower

Surface Area:

Volume:

6. The Great Pyramid of Giza

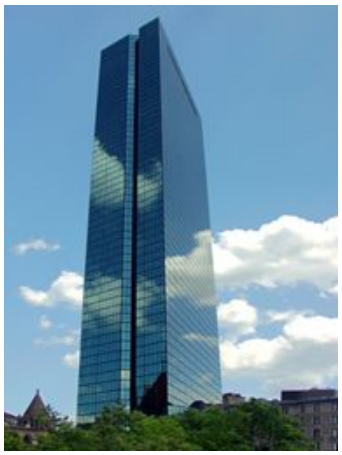
Surface Area:

Volume:

7. John Hancock Tower

Surface Area:

Volume:



8. Leaning Tower of Pisa

Surface Area:

Volume:

9. Chase Tower

Surface Area:

Volume:

10. **Tower Building**

Surface Area:

Volume:

11. **Corficolombiana Building**

Surface Area:

Volume:

12. The Pentagon

Volume:

13. Flat Iron Building

Surface Area:

Volume:

14. Use a search engine to find a building that is relatively cone, sphere, or cylinder shaped. Find and label the necessary dimensions that would be used to find the surface area and volume of that building. Next, use these dimensions to calculate the surface area and volume. Please name which building you chose and where it is located.

Success Criteria for Assessment

#'s 1-13	**5** The student provides the correct answer and the correct work	**4** The student provides a mostly correct answer and mostly correct work. Only a very minor procedural error was made or an error was made in regards to units.	**3** The student had the wrong answer but provided work that indicated they were very close to reaching the learning target	**2.5** Correct answer but completely incorrect work or no work provided	**2** Wrong answer and wrong work but the problem was fairly attempted	**1** Wrong answer was given and no work was provided	**0** No attempt was made
#14	**10** The student provides the correct answer and shows a deep understanding of the mathematical language and how to use it when explaining the formula used, procedures followed, and the answer.	**8** The student provides the correct answer but only shows an average understanding of mathematical language and how to use it when explaining the formula used, procedures followed, and the answer.	**6** The student provides the wrong answer but shows clear understanding of the mathematical language when explaining the formula, procedure, and answer to the class.	**5** The student provides the correct answer but struggles using mathematical language to explain the formula, procedure, and answer to the class.	**4** Correct answer was given but the student is unable to use mathematical language to explain the formula, procedure, and answer to the class.	**2** Wrong answer was given and the student was unable to use mathematical language to explain the formula, procedure, and answer to the class.	**0** No attempt was made

The assignment is out of 200 points. However, it is only going in the gradebook as a ten point assignment. The students' grades will be converted to a percent and then divided by 10 to get the grade that the student will have in the gradebook. For example, if a student got 183, I would take and get .915 which is 91.5%. Divide that by 10 and you get 9.15. So that student would get 9.15 out of 10. (Chances are I will round up to the nearest .5 point). This is a fun way to teach the students that the number of points an assignment is worth doesn't necessarily mean anything. It's the percentage that counts. They can see for themselves that no difference is made when entered in Skyward if I made the assignment out of 200 or if I made it out of 10.