High School
Algebra 1
Pythagorean Theorem/Trigonometric Ratios

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The Pythagorean Theorem is used in real life experiences. Using the Pythagorean Theorem we can calculate the diagonal of a room, find the height of a building given an angle that an individual is looking at the building, and we can calculate how far a ladder is leaning against building. The concept of the Pythagorean Theorem is important for students to learn in order to be able to use in their life in and outside of school.

The students will be learning about the six trigonometric ratios and the Pythagorean Theorem in an Algebra 1 course at a rural high school of 600 students and a class of 30 students. The textbook that will be used is an Algebra 1 book by Glencoe Mathematics. In the class, some students struggle with written word problems. The use of visuals will be in integrated in order to help the students meet the learning targets throughout the three day learning progression. The learning targets for day 1 are: Students will be to prove the Pythagorean Theorem by dissection; and students will be to use the Pythagorean Theorem to solve problems. The learning targets for day 2 are: Students will be able to know what side ratios are in right triangles; and students will be introduce and evaluate trigonometric ratios using the side lengths of a triangles. For day 3, the learning targets are: Students will able to use the Pythagorean Theorem to solve problems given two sides of a triangle; and Students will be able to use the trigonometric ratios and the Pythagorean Theorem to solve application problems.

The clusters for this learning progression are listed on the right. The learning targets for this three day progression are aligned to the Common Core Math Standards of: 8.G.B.6, 8.G.B.7, HSG.SRT.C.6, HSG.SRT.C.8. The learning targets are also aligned to four mathematical practices: MP1, MP4, MP5, MP6. (Please see on the right.)

First Cluster
Understand and apply the Pythagorean Theorem.

Common Core Math Standards
CCSS.MATH.CONTENT.8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse.
CCSS.MATH.CONTENT.8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Second Cluster
Define trigonometric ratios and solve problems involving right triangles.

Common Core Math Standards
CCSS.MATH.CONTENT.HSG.SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CCSS.MATH.CONTENT.HSG.SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Mathematical Practices
MP1: Make sense of problems and persevere in solving them.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.

The students will be beginning by proving the Pythagorean Theorem by dissection and then applying the Theorem to solve problems. The students will be able to prove by dissection the Pythagorean Theorem and will be applying the Pythagorean Theorem to solve problems. The Common Core Math Standards that are aligned with the beginning lesion are: 8.G.B.6 and 8.G.B.7. The mathematical practices that the students will demonstrate are MP1, MP5, and MP6. The students will need to be precise in their proof when writing it down or they will be confused later on. The students will be using a graphing calculator in order to complete the problems. The students will also be making sense of the problem in order to understand what the problem is asking. To being the lesson, the teacher will introduce the history of the Pythagorean Theorem. Allowing the historical background will help the students understand how it was created, who created it, and why it was created for. After the introduction of the topic, the teacher will move on to proving the Pythagorean Theorem by dissection. The teacher will scaffold the proof with the students. According to Wass and Golding, "Scaffolding provides assistance for a task so that students learn to do the task independently…" (p. 677). Scaffolding will help the students interpret and analyze the proof better. Shown on the side, Figure 1-1, is the proof that the class will be going to do. The proof helps the student to understand the conceptual understanding of how the Pythagorean Theorem is valid. The students will then use the Pythagorean Theorem to solve problems. The problems that the students will be working on will be developing the student's procedural fluency because there is the Pythagorean Theorem that the students will be using. The problems that the students will be doing are shown in Figure 1-2. The students are able to work individually or with the patterns that they are sitting with. Their partners allow for peer-tutoring to happen within the classroom. They are also able to ask the teacher for one-on-one help with the assignment. To accommodate for students, an extension on time will be allowed in order to have the students learn to the best of their capabilities. The worksheet can also be enlarged for students with visual needs. Then the

Figure 1-1

Proof by dissection:

Find the area of the diagram:

I. Area of the large square $= (a+b)^2$

II. Area of the small square including triangles$= 4\left(\frac{1}{2}ab\right) + c^2$.

Thus, $c^2 + 2ab = (a+b)^2$

$c^2 + 2ab = a^2 + 2ab + b^2$

Therefore, $c^2 = a^2 + b^2$.

Done.

Diagram:

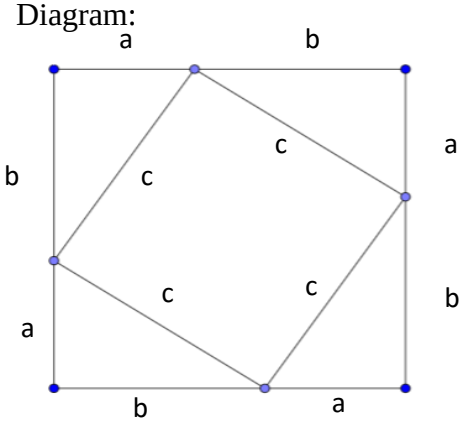
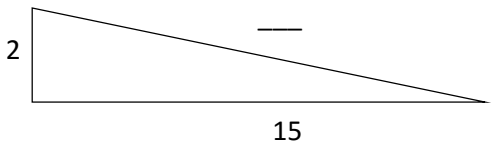


Figure 1-2

1. If one side of a triangle is 7 feet and the other side is 8 feet. Find the hypotenuse.

2.



3.

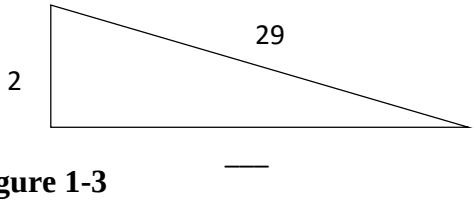


Figure 1-3

Find the length of a ladder that is 12 meters and is leaning against a 10 meter building. Howe far is the ladder from the building? See the diagram below. Show your work.

students will be given an exit task 10 minutes before class. In Figure 1-3, the exit task is shown. The exit task will be a formative assessment checking the understanding of the students overall ability to use the Pythagorean Theorem to solve real world problems. The exit task also help check the student's procedural fluency when doing the problems.

For the next lesson in the learning progression, the students will be able to know what side ratios are in right triangles. Students will be introduce and evaluate trigonometric ratios using the side lengths of a triangles. The Common Core Math Standard that are aligned to the learning target is SRT.C.6. The mathematical practices that the students will demonstrate are MP1, MP5, and MP6. The students will be making sense out of the problems and will be needing to be precise on the solutions. The students will also be needing to use the calculator in order to calculate the specific values for the lengths. To begin the less, the students will be introduce to the names of the sides of a triangle: opposite, adjacent, and hypotenuse. This small activity will allow the students conceptual understanding develop because they will be learning the basics of the name of the triangles. Then teacher then will lead the students to be introduce to the six trigonometric ratios that can students must be able to recognize which sides they are looking for when given a be used to find the lengths of a triangle given some information of a triangle. In Figure 2-1, shows the six trigonometric ratios that the students will be learning about. After the students learn about the trigonometric ratios, the students will be using the trigonometric ratios and be applying them to problems given different triangles with different lengths. The students will be demonstrating their procedural fluency skills as they work through their classroom work. In Figure 2-2, shows the problems that the students will receive. These problems will be used as informal assessment and will be used as participation points. It is understandable that not all students will not be able to get all the problems done in a timely manner. In order to accommodate for struggling students, they will need an extension on time. The students will be able to take it for homework and have it be seen by the teacher the next day for participation points. The

Figure 2-1

$\sin(\theta) = \frac{opp}{hyp}$ $\cos(\theta) = \frac{adj}{hyp}$

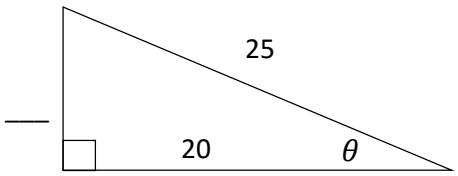
$\tan(\theta) = \frac{opp}{adj}$ $\csc(\theta) = \frac{hyp}{opp}$

$\sec(\theta) = \frac{hyp}{adj}$ $\cot(\theta) = \frac{adj}{opp}$

Figure 2-2

1. Given a ΔABC, with B as a 90˚ angle, angle C is 34˚. If $a = 4$, find the missing sides and angles.
2. Given a ΔABC, with B as a 64˚ angle, angle C is 23˚. If $a = 7$, find the missing sides and angles.
3. Given a ΔABC, with B as a 135˚ angle, $c = 3$, find the missing sides and angles.

Figure 2-3



$\sin(\theta) =$ $\frac{15}{25}$

$\cos(\theta) =$ $\frac{15}{20}$

$\tan(\theta) =$ $\frac{20}{15}$

$\csc(\theta) =$ $\frac{25}{20}$

$\sec(\theta) =$ $\frac{25}{15}$

$\cot(\theta) =$ $\frac{20}{25}$

Figure 3-1

_____ $= \frac{adj}{hyp}$ $\tan(\theta) =$_____

$\csc(\theta) = \frac{1}{(\quad)}$ $\sin(\theta) =$_____

_____ = _____=

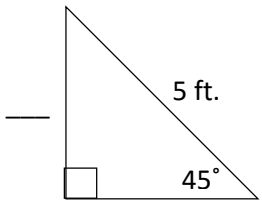
Bonus question: What is the Pythagorean Theorem? State the Theorem and what we use it for.

students can also work in groups in order to have peer-support and allow for understanding of the problems to be greater. This will allow the students to be able to have a better opportunity in reaching the learning targets. In addition, in Figure 2-3, the assessment is shown. The students will be completing a short match quiz towards the end of the lesson. The students will be matching the appropriate trigonometric ratios to their appropriate value given the side lengths of a triangle.

For the final lesson in the learning progression, the students will able to apply the Pythagorean Theorem to solve problems given two sides of a triangle. Students will be able to use the trigonometric ratios and the Pythagorean Theorem to solve application problems. The Common Core Math Standard that is aligned for the lesson is SRT.C.8. The students will be given a short quiz at the start of the class. The quiz contains the students of writing down the appropriate trigonometric ratios from a given triangle without using a notecard or notes. In Figure 3-1, shown on the right of the previous page, the assessment is shown. After about 5 minutes, the teacher will being with showing the students two examples of how to use the Pythagorean Theorem to solve the missing sides of the triangle. In Figure 3-2, on the next page, shows the problems that the class will work through. This activity allows the students to practice their procedural fluency. The students will be working on problems that consist of the students to use the Pythagorean Theorem and trigonometric ratios. The students will need to reason which trigonometric ratio to use to find the missing length and angle. In Figure 3-3, are example problems that the students will have to calculate the missing angles and sides of the triangles. The students will have the option of getting more time by having the assignment as homework. Then towards the end of the lesson, the students will the students will be given an application problem. The students will need to be able to complete the problem individually. The application problem is shown in Figure 3-4. The problem is a real life problem that will allow the students to recognize that mathematics is shown everywhere. The students will also need to use the Pythagorean Theorem and the trigonometric ratios students will be

Figure 3-2

1.



2.

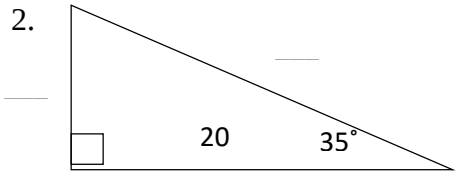
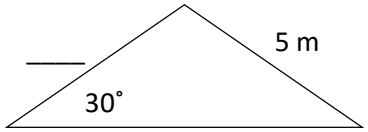


Figure 3-3

1.



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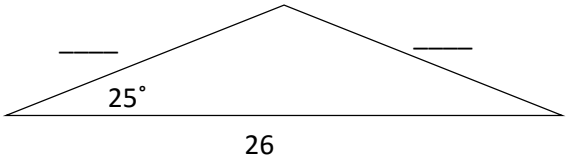

Figure 3-4

The Pythagorean Theorem is used calculating unknown lengths. If the distance to first base is 27 m and the distance from first to second is 27 m. Then how far do I have to run if I start at home base to get to second base, if I run straight across?

What are the six trigonometric ratio's in correspondence to the values you calculated in the picture? The length of side a and b are 27 meters. Find side c's value. Show your work below.

$\sin(\theta) =$ $\cos(\theta) =$ $\tan(\theta) =$
$\csc(\theta) =$ $\sec(\theta) =$ $\cot(\theta) =$

the four mathematical practices of MP1, MP4, MP5, and MP6. The students will be solving problems to model mathematics. They are precise in their calculations because the measurements need to be precise. The students will also be making sense of the problem, especially the exit task question because it has the student think using real world situations.

Furthermore, there will be assessment given in each lesson of the learning progression to measure the student's mastery towards the learning targets. There will also be accommodation for students such as an extension of time for assignments, peer-support, peer-teaching, working individually, and working in groups. The instruction in the classroom will allow a better opportunity for the students to meet the learning targets. The formative assessment throughout the learning progression will be short and directly testing the main points that the students should be able to do by the end of the class or by the next day of class. The formative assessments also help the teacher check for understanding in the student's comprehension. The formative assessments must be implemented before and after lectures in order to check and see if more instructional time is needed for the students to master the learning targets for each lesson. Checking for understanding allows for the teacher to move on with a lesson or maybe revisit a part of the lesson that all students are struggling one. Thus, the formative assessment will be informal and formula. The quizzes will be formal and will have feedback for the students. The feedback is the most important because the students are learning from the feedback given. This also allows the students to make their own improvement in their learning if they want to learn more. The homework will be graded informally and will be used as participation points if the student has tried the homework.

Overall, the students need a balance of learning and practicing how to do the problems. This learning progression of the Pythagorean Theorem is not an easy progression. If a student misses a day of class, it will be hard for the student to learn what was learned from the day before. In this learning progression, it is important for the students to attend class and ask questions when they are confused.

There are a good amount of assessments that allow the teacher and the students to know whether they need more practice and time to master the learning targets of the learning progression. Thus, the teacher must be able to accommodate and adapt to all the learning styles and mathematical background that will be in the classroom learning about the Pythagorean Theorem and the trigonometric ratios.

Works Cited

Wass, Rob, and Clinton Golding. "Sharpening A Tool For Teaching: The Zone Of Proximal Development." *Teaching In Higher Education* 19.6 (2014): 671-684. *Academic Search Complete*. Web. 10 Feb. 2016.