**High School: Functions**

**Building and Interpreting Exponential Functions**

This learning progression would take place in a 10th/11th grade Trigonometry classroom. The progression is aligned with the McDougal Littell Algebra 2 textbook, specifically chapter 8 of the text. Not only does this series of activities follow a textbook, it also addresses multiple Common Core State Standards regarding building and interpreting functions. In this learning progression, students will be building functions to model real-world situations (CCSS.MATH.CONTENT.HSF.BF.A.1, CCSS.MATH.CONTENT.HSF.LE.A.2) and they will also be graphing these functions using transformational techniques (CCSS.MATH.CONTENT.HSF.BF.B.3).

A majority of this lesson will be direct instruction, in order for students to understand the concepts before using them to solve problems. As students become more familiar with the topic, the teacher can introduce story problems to test the student’s ability to model with mathematics (MP4). Students will need to truly understand exponential growth and decay before they can reason abstractly and quantitatively (MP2) to make sense of story problems and persist in solving them (MP1).

Once students have a firm understanding of the mathematics, the teacher can move from a direct instructional method and into more student-led discovery based activities. In this way, students will have the opportunity to work with their peers to expand on the concepts the teacher has presented to them. Group work is incredibly beneficial when dealing with new material because students get to prove their understanding by explaining the concepts to struggling peers. Students also get a chance to ask their peers questions, which can be beneficial for those uncomfortable speaking in front of the class.

**Building and Interpreting Exponential Functions**

Instead of starting at the beginning of a chapter in the textbook, this learning progression begins in the middle of a chapter. Students should have a firm grasp of the basic concepts regrading exponential growth and decay, as well as the basic definition of logarithms. Students should also have a basic understanding of the number *e*. This learning progression focuses on building and interpreting real-world situations, not basic definitions and procedures.

The first day would focus on story problems involving exponential growth and decay with specific focus on compounding interest. On this day, students will be introduced to the basic form of exponential growth and decay models, as well as the formula for compounded interest. It is important to clearly define the variables in such a way that students will be able to pick out values from a scenario and input them in the correct formula. After clearly labelling the variables, it is best to walk through an example or two in order to cement understanding. Use the benchmark assessments to the right to determine student understanding (BM1, BM2).

BM1 – A smartphone can be purchased for $600 but decreases in value 29% every year. Estimate the value of the phone after 3 years. When will the phone be worth $100?

BM2 – In January there were 15 members in a club. Over the next 5 years the number of members increased by 60% per year. Determine a model and estimate the year the club had 25 members.

Once students are secure in their understanding of exponential modeling, it is time to begin compounded interest. The concept follows from exponential growth with the introduction of the variable n, the number of times the interest is compounded per year. A good way to teach this concept is to present different scenarios, each compounded a different number of times during a year, and asking students to pick the best one and explain their reasoning. In this way, students have to use their newly gained knowledge as evidence for their claim. In other words, they have to make sense of a story problem and persist in solving it (MP1). Not only that, but students are modeling a situation with mathematics and using that model to make a decision (MP4).

The next day focuses on story problems with logarithms and the number *e.* As stated before, students should be familiar with basic concepts regarding logarithms and the procedure behind solving an equation that has a log or natural log in it. This lesson will show students how to approach logarithmic story problems and also give students more practice with working with logarithms.

Example questions, as well as practice problems, should use a variety of scenarios, though it may be hard to find different scenarios that use logarithmic functions or the number *e.* Some common scenarios to use for modeling examples include: modelling the magnitude of an earthquake, modeling the decay of certain particles and understanding altimeters.

Another concept that should be introduced in this lesson is the logistic growth function. Now, using the logistic growth function, students will be able to model different populations and interpret what the model say about a function (HSF.BF.A.1, HSF.LE.A.2). Different problems can be posed to check for, and even build upon, understanding (BM3).

This day in the learning progression will start with direct instruction so students can figure out how to work with logarithms and the number *e*. After an understanding is reached, then the lesson will be more discovery based, where students get to explore with their skills to model a variety of situations.

The last day of instruction in the learning progression is entirely student based. The lesson involves a group activity where students can compete with their peers to demonstrate their understanding of the concepts. Certain questions similar to examples done in class, including benchmark assessments, will be presented to the students and they will work in groups to complete them. The group with the most questions correct at the end of the lesson receives some sort of reward.

#### MP1: Make sense of problems and persevere in solving them.

#### MP4: Model with mathematics.

**HSF.BF.A.1**: Write a function that describes a relationship between two quantities.\*

**HSF.LE.A.2**: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

BM3 – A population of bacteria is growing in a petri dish. The area of the bacteria can be modeled by the equation:

$$A=\frac{49.9}{1+134e^{-1.96t}}$$

where t is the elapsed time in days. Graph the function and describe what it tells you about the growth of the bacteria colony.

For this activity, students get hands on experience solving problems while engaging in healthy competition with their peers. Adolescents seem to react favorably when an activity is structured like a game, especially if there is some sort of reward on the line. Camouflaging an educational activity as a game can also inspire some students, who would otherwise slack off, to get involved with the activity.

Group work is also incredibly beneficial for students as it allows them to work in a low stress environment. Sometimes, when it comes down to asking a question in front of the class, students will refrain in order to avoid looking foolish. When working in a group, students can ask their peers questions they would normally keep to themselves. Furthermore, explaining concepts to their peers who need help reinforce understanding, and ensures that the student really knows the topic.

In conclusion, this learning progression builds on the topics of exponential functions, logarithms, and the number *e*, rather than introduce the concepts. Students know the basic procedural concepts, but now have the opportunity to work with the material on another level.