High School: Geometry

Right Triangles and Trigonometry

This learning progression was designed primarily for a slower pace 9th-grade geometry course. The Common Core State Standards that will be satisfying are from two different domains. The first two standards come from the cluster titled, "Understand and apply the Pythagorean Theorem," these are 8.G.B.6 and 8. G.B.7. The second two content standards come from the cluster titled, "Define Trigonometric ratios and solve problems involving right triangles," which are HSG.SRT. C.6 and HSG.SRT. C.8. In this course, students focus on mastering 8th grade standards as they slowly incorporate high school content standards. Throughout this learning progression, students will focus on four mathematical practices which are MP1, MP3, MP4, and MP6.

The curriculum these students are going through comes from the 2011 Holt McDougal Larson Geometry textbook. For this learning progression, students are beginning a brand-new unit on right triangles and trigonometry. Specifically, applying the Pythagorean Theorem and its converse to classify types of triangles along with students being able to identify similar triangles and write ratios.

The central focus of this learning segment is an investigation of side lengths and angles in triangles to be able to find missing side lengths and being able to classify different types of triangles. The purpose of this content is to give students the mathematical tools to use to work with all types of triangles, especial right triangles. The underlying concepts are right triangles, Pythagorean theorem and its converse, and similarity of triangles. The simple knowledge in this learning segment includes the

COMMON CORE STATE STANDARDS

Understand and apply the Pythagorean Theorem

CCSS.MATH. CONTENT.8. G.B.6:
Explain a proof of the Pythagorean Theorem and its converse.
CCSS.MATH. CONTENT.8. G.B.7:
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in the real world and mathematical problems in two and three dimensions.

Define Trigonometric ratios and solve problems involving right triangles.

CCSS.MATH.CONTENT.HSG.SRT. C.6
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CCSS.MATH.CONTENT.HSG.SRTD.8:
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied *problems.*

MATHEMATICAL PRACTICES

CCSS.MATH.PRACTICE.MP1
Make sense of problems and persevere in solving them.
CCSS.MATH.PRACTICE.MP3
Construct viable arguments and critique the reasoning of others.
CCSS.MATH.PRACTICE.MP4
Model with mathematics.
CCSS.MATH.PRACTICE.MP6
Attend to precision.

vocabulary relating to triangles, such as right triangles, and the definitions of all components to the Pythagorean Theorem and its converse. The procedure of this learning segment is for students to learn how to apply the Pythagorean Theorem to find the length of the third side in a right triangle, and then being able to use the Converse of the Pythagorean Theorem to decide if the three given side lengths form an acute, right, or obtuse triangle. After being able to find missing side lengths and being able to classify triangles students will be able to explore ratios of side lengths of similar triangles. All leading to students developing enough mathematical reasoning to apply their knowledge to real-world mathematics. Throughout each lesson, I will use the data on prior academic learning and disposition from my observations and the series of entry tasks students turned in to support my students' learning. I sequenced my learning targets to start with familiar learning targets and branched to learning goals that depended on a mastery of the previous ones. Breaking down the learning objectives is beneficial to students with need of support or accommodations, since focusing students' attainment of immediate goals, such as getting today's problem correct increases student self-efficacy according to A. Wade Boykin and Pedro Noguera's book titled "Creating the Opportunity to Learn."

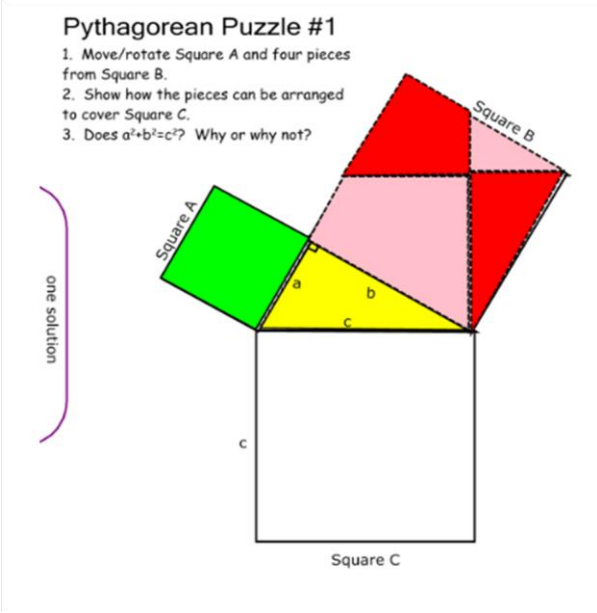
In Task 1, students will receive a handout activity puzzle where the students will use problem-solving skills to prove to themselves the Pythagorean equation is true. Students will cut out squares A and B to rearrange the pieces to prove that the pieces fit in square C, proving that $a^2 + b^2 = c^2$. Once the students have rearranged the puzzle pieces the students will use their mathematical reasoning to justify why $a^2 + b^2 = c^2$. The students will explain to their partners their reasoning of why the equation is true and how they made the connection between

LESSON 1

Learning Target

I can apply the Pythagorean Theorem to find missing side lengths, identifying the correct variable missing (hypotenuse or leg).

Task 1:



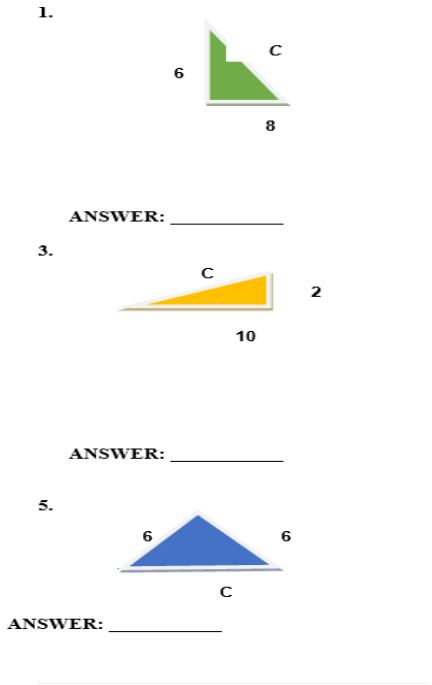
Guiding questions:

- Does $\boldsymbol{a^2 + b^2 = c^2}$? Why or why not? (Yes, our activity proves this)
- When we say a "square" plus b "square" equals c "square," what do we mean? (What we mean is the area of the actual square made on each side of the right triangle)
- What is the purpose of the Pythagorean Theorem? (To find the missing length of the side of a right triangle)
- How can I use the Pythagorean Theorem to find the missing length of a right triangle? (Use the formula)
- How does the length of the hypotenuse compare to the length of a leg? (it is longer)

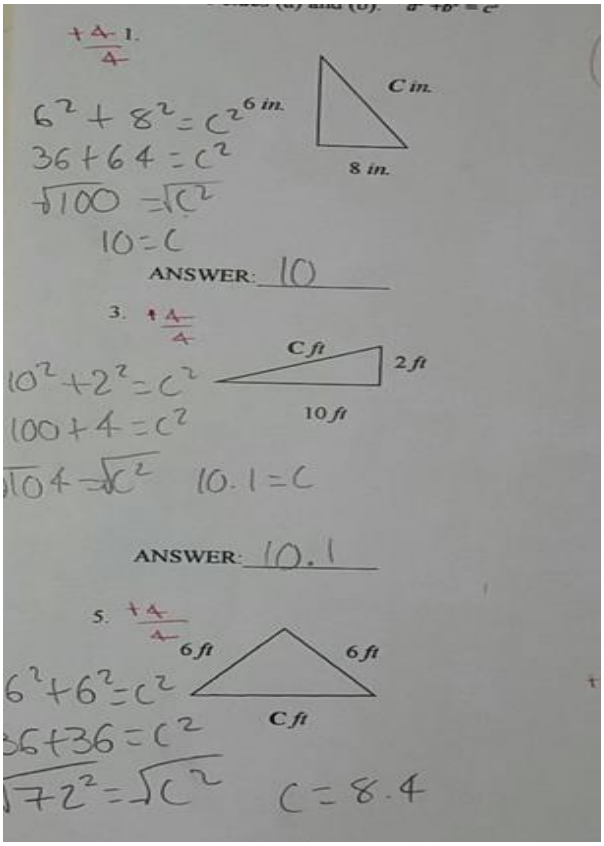
the concept presented in the diagram and the equation. Students must be able to be able to explain to their classmates in their preferred language how the puzzle proves the equation. This will be done as a presentation to the class as an assessment of their understanding of the Pythagorean equation. This Task will address content standard 8.G.B.6 as the students will be able to explain a simple proof the Pythagorean Theorem. Along with this student will incorporate all four mathematical practices included in this learning practices. MP1 will be covered as students must make sense of the puzzle and persevere in solving the puzzle. Once the students have been able to solve the puzzle they will work on MP3 as they construct their argument to explain why it is the puzzle proves the Pythagorean equation. The teacher will monitor the activity by observing and posting guiding questions on the board for students to discuss as they work through the puzzle. Students will have the opportunity to express their understanding through whole-class discussion in which selected students will explain to the class their findings from the activity. During their presentation students, will use MP4 and MP6 as they will model with mathematics where the equations comes from attaining to precision.

Task 2 will entitle the completion of a handout that students will have the opportunity to work through multiple times. Students will receive a handout with a series of pre-drawn triangles to solve for the missing side lengths using procedural fluency to work through solving for a specific variable. This task will address CCSS.MATH. CONTENT.8. G.B.7 where students would use conceptual understanding in their handout when they are able to recognize the correct values to substitute into the Pythagorean equation. This means able to identify the hypotenuse and the legs of a right

Task 2:



STUDENT WORK SAMPLE



triangle. This activity will be the foundation for students to be able to apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world problems. To conclude the handout students will be asked to present a solution to a random problem on the handout to the class as a quick assessment of their understanding to complete Lesson 1 to the learning progression. During this task students, will again attain to all four mathematical practices as they complete the handout individually and then join a whole class discussion to critique others reasoning of the solutions and work to come to a common agreement to answers.

Lesson 2 will begin with a brief guided practice for students receive the foundation to be able to use the converse of the Pythagorean Theorem. This lesson is designated as an extension lesson for the unit as it does not directly apply to the CCSS for the unit. For task 3, students will continue to develop procedural fluency from lesson one in that they will continue to write and solve equations using the Pythagorean equation to solve for missing lengths in right triangles. The focus of the learning target for Task 3, is the development of procedural fluency. Students will be able to use the converse of the Pythagorean theorem to be able to classify acute, right, and obtuse triangles. For students to be able to classify, different triangles using the converse of the Pythagorean theorem they must understand the different classification properties. Students will demonstrate conceptual understanding when they are able to correctly recognize what type of triangle they have in their handout. For this lesson students, will be given a handout similar to the one they received in Task 2. In this handout students, will determine what types of triangles they have and be able to connect the procedural fluency they used when using the converse of the Pythagorean Theorem and just using the

LESSON 2

Learning Target:

I can use the Converse of the Pythagorean Theorem to decide if the three given side lengths form an acute, right, or obtuse triangle.

Student Outcomes

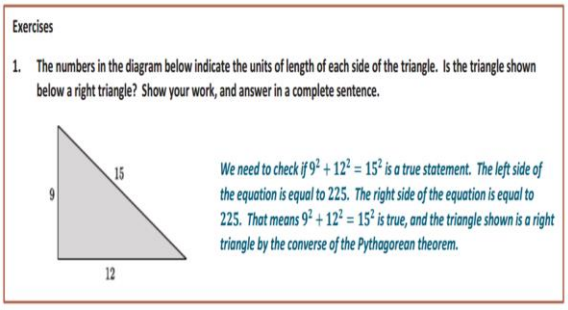
Students illuminate the converse of the Pythagorean theorem through computation of examples and counterexamples.

Students apply the theorem and its converse to solve problems.

Task 3:

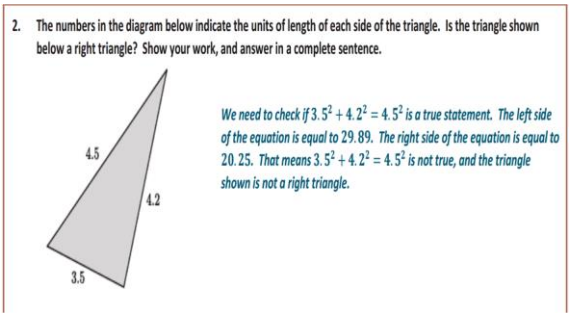
Converse of the Pythagorean Theorem: If the lengths of three sides of a triangle, a, b, and c, satisfy $a^2 + b^2 = c^2$, then the triangle is a right triangle, and furthermore, the side of length c is opposite the right angle.

If $a^2 + b^2 > c^2$, the triangle is acute, if $a^2 + b^2 = c^2$then the triangle is right and if $a^2 + b^2 < c^2$then the triangle is obtuse.


Exercises

1. The numbers in the diagram below indicate the units of length of each side of the triangle. Is the triangle shown below a right triangle? Show your work, and answer in a complete sentence.

We need to check if $9^2 + 12^2 = 15^2$ is a true statement. The left side of the equation is equal to 225. The right side of the equation is equal to 225. That means $9^2 + 12^2 = 15^2$ is true, and the triangle shown is a right triangle by the converse of the Pythagorean theorem.


2. The numbers in the diagram below indicate the units of length of each side of the triangle. Is the triangle shown below a right triangle? Show your work, and answer in a complete sentence.

We need to check if $3.5^2 + 4.2^2 = 4.5^2$ is a true statement. The left side of the equation is equal to 29.89. The right side of the equation is equal to 20.25. That means $3.5^2 + 4.2^2 = 4.5^2$ is not true, and the triangle shown is not a right triangle.

Pythagorean Theorem to solve for missing lengths when appropriate. This handout will be completed in teams of four in which each member of the group will have a task. Member 1 will be the spokesperson who will present a solution to the class when called upon. Member 2 will be the recorder and will write on the whiteboard the steps to solve each problem. Member 3 will be the handout recorder who will be the one who writes the solution on the handout to turn in for credit at the end of the activity. Finally, member four will be the understanding coordinator who will make sure the calculations are checked, and mathematical reasoning is justified. These jobs will be rotated until all the problems on the handout have been completed with class discussion.

For Task 4, students will receive a handout with some pre-drawn triangles, in which students will need to separate the triangles to find corresponding sides like shown in the Task's handout examples. Students will use the side lengths and angles provided of all the triangles to determine whether any triangles are similar. When the students have recognized which of the triangles are similar, they will write ratios using two side lengths at a time from the three known side lengths. Once the students have written their similarity ratios of the right triangles provided they will be asked how the ratios for similar triangles compare, these responses will be recorded as the teacher approaches each student as they work independently on their handout. This question will lead them to determine that the ratios of corresponding side lengths for similar triangles are equal. Having students come to this conclusion through their handout will address HSG.SRT. C.6, which states, "Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles." Students will demonstrate this by using mathematical

LESSON 3

Learning Target:

I can identify similar triangle and write proportions for pairs of side lengths in given right triangles.

Student Outcomes

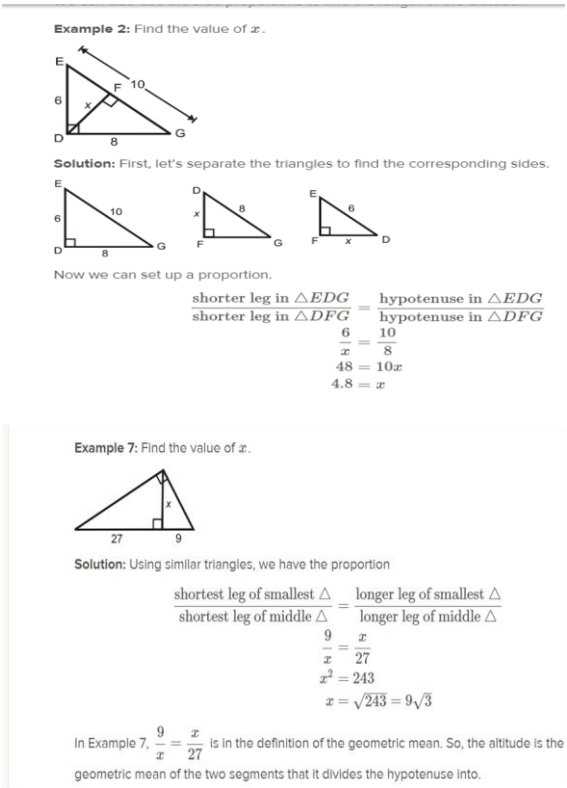
- Identify similar triangles inscribed in a larger triangle.
- Evaluate the geometric mean.
- Find the length of an altitude or leg using the geometric mean.

Task 4:

Guiding questions:

1. If two triangles are right triangles, does that mean they are similar? Explain.
2. If two triangles are isosceles right triangles, does that mean they are similar? Explain.
3. Solve the ratio: 3/x = x/27.

Handout Examples:



Example 2: Find the value of x.

Solution: First, let's separate the triangles to find the corresponding sides.

Now we can set up a proportion.

$$\frac{\text{shorter leg in } \triangle EDG}{\text{shorter leg in } \triangle DFG} = \frac{\text{hypotenuse in } \triangle EDG}{\text{hypotenuse in } \triangle DFG}$$

$$\frac{6}{x} = \frac{10}{8}$$

$$48 = 10x$$

$$4.8 = x$$

Example 7: Find the value of x.

Solution: Using similar triangles, we have the proportion

$$\frac{\text{shortest leg of smallest } \triangle}{\text{shortest leg of middle } \triangle} = \frac{\text{longer leg of smallest } \triangle}{\text{longer leg of middle } \triangle}$$

$$\frac{9}{x} = \frac{x}{27}$$

$$x^2 = 243$$

$$x = \sqrt{243} = 9\sqrt{3}$$

In Example 7, $\frac{9}{x} = \frac{x}{27}$ is in the definition of the geometric mean. So, the altitude is the geometric mean of the two segments that it divides the hypotenuse into.

reasoning to classify similar triangles and realize that the ratios they made are equal, discussing with partners their thinking process as they work. Along with this student will also address, HSG.SRTD.8, "Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems." Students will address this after they have been able to grasp the concept of similar triangles.

To conclude the unit students will receive a cumulative handout that will bring all four major tasks together. This handout will allow the students to use conceptual understanding when they can correctly form their Pythagorean equation to solve for a missing side length or classify a triangle. This will be done individually to provide more concrete data of their overall understanding of the big concepts needed to meet the CCSS and the mathematical practices incorporated in this learning progression. Problems on this handout will include real world situations to allow students to deepen their understanding. All three tasks require students to build knowledge to be able to read and interpret real life scenarios. They will need to be able to represent their mathematical thinking in words as well as pictorially.

For this learning progression, I will use two main instruction methods throughout each lesson. I will begin the lessons using explicit/direct instruction in which students will receive the foundation to be able to complete each task successfully. Then to deepen the students' understanding by completing each task using cooperative learning which will lead to the assessment cycle I will use throughout each lesson to analyze student learning and guide their instruction.

For this, I would be using two main formative assessment techniques which are listening to

FORMATIVE ASSESSMENT PROCESS AND INSTRUCTION METHODS

Cumulative Handout Example:

Buying a Suitcase: Mr. Harry wants to purchase a suitcase. The shopkeeper tells Mr. Harry that he has a 30 inch of suitcase available at present and the height of the suitcase is 18 inches. Calculate the actual length of the suitcase for Mr. Harry using Pythagoras' theorem. It is calculated this way:

$$(18)^2 + (b)^2 = (30)^2$$

$$324 + b^2 = 900$$

$$B^2 = 900 - 324$$

$$b = \sqrt{576}$$

$$= 24 \text{ inches}$$

Instructional Methods:

- Explicit Instuction/Direct Instruction: involves teaching skill or concept in a highly structured enivironment using clear, direct language.
- Cooperative Learning: Involves students working together in small mixted-ability groups to maximize everyone's learning.

Foramtive Assessment Techniques:

- Listening to students' response: Walk around the classroom and observe students as they work to check for learning. Strategies include: anecdotal records, conferences, and checklist.
- Examining student work: Collect information to help precede in upcoming lesson.

students' response and examining student work. Types of questions I would ask my students to evaluate their understanding is "How would you convince some one that the Pythagorean equation works using your puzzle?" "Where do the pieces come from?" "Explain your reasoning." Then when examining student written work, I would have to ask myself questions like, "What does the collection of work suggest about next instructional steps?"

Throughout this I will use three main tasks as a formative assessment technique to support student learning of the CCSS Math as students will be presenting to the class their solutions and how they worked through each step which will provide me with data on areas that students need more clarification to master the CCSS along with each mathematical practice. Each task allows students to work through problems that connect to CCSS individually and has the students later share in their group answers and process through their work using the mathematical practices mentioned previously. This process will allow the students to work through problems multiple times. Ideally, each time the students work through the problems and explain their reasoning more misconceptions will be clarified and as the teacher, I will be able to pick up on areas that need special attention that will be covered in class.

Ideally, through the process, I have selected to complete the major tasks clarifying and motivating students' learning will occur as a result. Students will have the opportunity to work on problems individually first which will allow students to self-assess and pinpoint areas that they need clarification. Second, the students will break up into their groups and compare/work through problems, and this will motivate students to work together and try to clarify their areas of misunderstanding. Third when the students come up and present their solutions with justification for their answers students will be able to have a third opportunity to review receive clarification on areas that were not touched upon at the beginning of the assessment process. Lastly, I will observe and examine student work and cover any areas that I feel students need more clarification to master the CCSS. Throughout each step as students can review and explain problems multiple times their motivation will increase as there will always be a problem in which each student will be able to help another classmate understand a concept.

For special populations of students such as my ELL students, in every task I will provide translations as well as allow students to speak and present their work in their preferred language. This will be a given in any activity we do in class, students will always know that these supports are available for them to use at any given time throughout the school year. I feel that allowing this will make the students feel more confident in their abilities and make students more willing to present their work to the class. I also understand that presenting especially in math is a difficult task for many students, in special cases when I feel it is appropriate I will allow students to present solutions to partners to relieve some of the pressure. This making it so I can collect more accurate data over student understanding of the CCSS.