Common Core Standards to meet:

## Experiment with transformations in the plane

## CCSS.MATH.CONTENT.HSG.CO.A. 2

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

CCSS.MATH.CONTENT.HSG.CO.A. 3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

CCSS.MATH.CONTENT.HSG.CO.A. 4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

## CCSS.MATH.CONTENT.HSG.CO.A. 5

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Mathematical Practices (MP)

CCSS.MATH.PRACTICE.MP1:Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2: Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4 :Model with mathematics.

CCSS.MATH.PRACTICE.MP5: Use appropriate tools strategically.

## High School Geometry: Experiment with transformations in the plane

The present learning progression is designed for a $10^{\text {th }}$ grade geometry classroom. The textbook used is in this classroom is Geometry, Grades 9-12: Mcdougal Littell High School Math, $10^{\text {th }}$ edition. The Common Core State Standards that will be satisfied are HSG.CO.A2, HSG.CO.A3, HSG.CO.A4 and HSG.CO.A5. The math practices that students will make use are MP1, MP2, MP4 and MP5.

To meet the common core state standards, students will complete selected activities from the textbook that include understanding basic transformation vocabulary: reflections, rotation, translation and glide rotations; rotate shapes and equations of graphs around a line; compose two or more transformations together; explain any figures rotational symmetry; and define rotations using coordinate points and rotate points and shapes around a center of rotation on a graph. In this learning progression there will be a total of four lessons extended over 5 days of work. Formative assessment will be used during instruction by asking students to rate their understanding, share their answers in group discussions and ask/answer questions. Summative assessment will be activity sheets, exit slips and final assessment project.

By the end of this unit, students will complete a project base assessment that included the use of technology. For this project, students will select their favorite emoji and then they accurately translate it, reflect it and rotate it in the plane. All they need to do is document all the transformation rules and labels correctly, and answer three questions to show their understanding of the transformation rules.

## Lesson 1 Reflections

CCSS.MATH.CONTENT.HSG.CO.A. 2
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

## Example 1.



Refer to the diagram above.
Assume that $Y$ is the reflection of $X$ in line $m$, and $Z$ is the reflection of $Y$ in $n$.

1) Write a paragraph proof showing that distance $O X$ is equal to distance $O Z$.
2) How is related to Write a paragraph proof to defend your answer. (Assume is an acute angle with $Y$ in its interior, as shown.)

## Lesson 2 Rotation

## CCSS.MATH.CONTENT.HSG.CO.A. 3

Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Lesson 1 will introduce the students to the concepts of reflections, and representations of this transformation for different shapes (HSG.CO.A2 and HSG.CO.A4). The mathematical practices used in this lesson are MP1 and MP2. The beginning of the lesson, I will ask students to draw their favorite emoji in a paper and cut it. The objective is for students to identify and be able to reflect figures in the Cartesian plane. At the beginning of instruction, I will explain the new shapes that can be formed when we use slides, flips and turns of figures. At this point, I will give them the mathematical definition of a reflection and lines of symmetry. Then I will ask them to go to the board (there will be a Cartesian plane there already drawn) and do a reflection of the figure they cut. Then the rest of the class will have a short discussion on whether or not that was a rotation or not. This will help students discourse and see the concept with a real object. This activity will be the formative assessment, if students are able to explain their understanding and perform they rotations correctly then they are ready for the next activity. By the end of instruction, students will be able to complete the activity handout (benchmark assessment) that includes construction reflections of given figures, answering questions using a diagram (example 1) and finding the lines of symmetry of different polygons.

Lesson 2 will introduce students to the concept of rotations (HSG.CO.A.3). Students will also make use of mathematical practices MP2 and MP4. For this lesson as an introductory activity, I will draw a figure in the board along

## Example 2



Find the coordinates of $Q^{\prime}$ in terms of $a, b$, $x_{0}$ and $y_{0}$.

Find the coordinates of $R^{\prime}$ in terms of $a, b$, $x_{0}$ and $y_{0}$.

If the point $(5,3)$ is rotated $90^{\circ}$ counterclockwise about ( 0,0 ), what are the coordinates of the image point?

If the point $(2,-3)$ is rotated $90^{\circ}$ counterclockwise about ( $-3,7$ ), what are the coordinates of the image point?

## Lesson 3 Translations

CCSS.MATH.CONTENT.HSG.CO.A. 4
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Example 3.


Draw the image of PQRS after each translation
$(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}+1, \mathrm{y}-4)$
$(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}-1, \mathrm{y}-3)$
with its reflection. I will ask students to describe the reflection they see. This will help me assess their understanding of the previous material. After this activity, I will explain the mathematical definition of rotations and how they affect shapes. Before students work individually in their assignment, I will perform various rotations and ask them to name the rotation (clockwise, counterclockwise, $90^{\circ}, 180^{\circ}, 270^{\circ}$ etc.). This activity will be used as a formative assessment, if students give the right answers then they will be ready to worn on their handout. The handout includes applying their knowledge about rotational symmetry for different polygons, construct rotations of given points and lines and answer questions using a diagram (example 2). This activity will be then benchmark assessment for this lesson. If students are able to complete $80 \%$ of the handout correctly then they are ready to move into the next lesson, transformations. The last activity will be a critical thinking questions that will be used as an exit slip. Students need to explain how a compass and straightedge can be used to construct rotations and translations.

Lesson 3 will introduce student to the concepts of translations (.HSG.CO.A.4). Students will also make use of mathematical practices MP1, MP2 and MP4. At the beginning of instruction, I will ask give students an example that includes a figure and its translation. then I will ask students to write a description of the translation compared to the original figure. This will help students to inquire and compare answers with others. Then I will give students the definition of translation and work with them some examples before they can complete problems from the book. For

## Lesson 4 Glide Reflections

CCSS.MATH.CONTENT.HSG.CO.A. 5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Example 4


Use the information above to sketch the image of $A B C$ after a glide reflection.
$A(-1,-3) \quad B(-4,-1) \quad C(-6,-4)$
Translation: $(x, y) \rightarrow(x+10, y)$
Reflection: in the $x$-axis.
students to meet the benchmark assessment, they need to demonstrate a firm and clear understanding of all the three different transformations they have learned. The benchmark assessment in their lesson is working some book problems that will include identifying translations and describing them; answering questions about a diagram and performing different translations using the Cartesian plane and a polygon (example $3)$.

Lesson 4 will introduce the students to glide reflections (HSG.CO.A5). In this lesson, students will use MP1, MP2 and MP4. The objective of this lesson is for students to identify glide reflections in a plane and represent transformations as compositions of simpler transformations. To start this lesson, I will ask students to complete a translation and a reflection of a figure. Then I will give students the definition of glade reflection and I will choose one of their examples to do a glade reflection. After this I will ask students to explain why they see. This will be formative assessment, if students answer something similar to a glide reflection being a compositions of a translation and a reflection, the next activity will be presented. The activity handout has different problems that ask students to describe transformations using the correct notation and to find the equations after glides reflections have been applied (example 4). For students to meet the benchmark assessment, students need to be able to distinguish a rotation, translation and reflection from one another as well as learning how to compose two or more together. If students complete their activities correctly, then they have meet the objectives of the lesson.

## Final Project

CCSS.MATH.CONTENT.HSG.CO.A. 5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

CCSS.MATH.PRACTICE.MP1: Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2: Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4 : Model with mathematics.

CCSS.MATH.PRACTICE.MP5: Use appropriate tools strategically.

Students will be graded based on
Well documented transformation
Correct transformation
Correct labeling a notation
Attempt to neat

The final assessment for this lesson is a short project, where students need to select their favorite emoji and apply at least 2 reflections, translations and rotation. Students need to document all the transformations and label them correctly. Additionally, then need to complete three given questions: When a point is reflected in the x -axis, how are the coordinates if the image related to the coordinates of the preimage? What is a center of rotation? What attributes of shapes are preserved by glide reflection? This questions will help me assess students' understanding of the concepts rather than just showing procedures.

