High School: Calculus
A Beginning Look at Calculus

This learning progression is for a high school calculus classroom, primarily for 11th and 12th graders. This mathematics class is one of the highest level, in which students want to be in class and want to learn. In this classroom, the majority of these students do not have any IEP's nor 504 plans, but there are a couple of students whom I need to make accommodations, based on their needs. There is a student who has hearing impairment, and who is an English Language Learner. This is the first unit of Calculus, where students will be introduced to the two goals of calculus. The Common Core State Standards that it will be satisfying are the following: HSF.IF.A.1, HSF.IF.A.2, and HSF.IF.B.4.

The textbook that students will be using in this class is Calculus, Volume 1, Chapter 1, by Dietiker, Kysh, Sallee, and Hoey, published in 2010. Before this lesson, students reviewed over some key concepts of pre-calculus. Some of the topics that students reviewed were the different types of function transformations, such as vertical translation, horizontal translation, vertical reflection, horizontal reflection, vertical dilation, and horizontal dilation. Students have also learned piecewise functions, compositions, inverses, even & odd functions, domain & range, and horizontal and vertical asymptotes.

The learning progression will start first by students developing concepts of slope and slope functions. Students will show particular functions change by examining finite differences. Because this review was conquered only a matter of weeks ago, I will guide my students throughout the learning progression to understand the relationship to what they already know, to what they will learn in this unit. This is very important to students to see the relationship because not only they are able to apply the strategies, but they are reinforcing what they have learned and apply it to different settings. Meanwhile, the central focus of the lesson progression presented here will be understanding how to find the tangent to a curve at a point, as well as finding the area under a curve between two points. The students will then discover how limits work.

Common Core State Standards

Content Standards

1) CCSS.MATH.CONTENT.HSF.IF.A.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

2) CCSS.MATH.CONTENT.HSF.IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3) CCSS.MATH.CONTENT.HSF.IF.B.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

At the beginning of the lesson progression, the students will be given figure 1, which can be seen in the left margin, and the equation $f(x) = 5 - x$. Students will examine the graph of the function. To ensure that the Common Core State Standards HSF.IF.A. and HSF.IF.A.2. are met, I will ask students to make a T-table to write down the x-values on the left side and f(x) on the right side of the table to denote the output of f corresponding to the input x. After plotting in the points on the graph, students will see how the function turned out to be. Then, I will shade a specific part of the graph. In addition, students will find the shaded region using geometry. The notation $\mathrm{A}(f, 0 \le x \le 5)$ will represent the area between the function, and the x-axis over the interval from x=0 to x=5. This should be a review from the past week.

Each student will work with their peer, who sits beside them. I made accommodations to the English Language Learner (ELL) student by assigning him to sit beside a bilingual student. Students will have discourse over how to find the area of the shaded part. This will help the ELL student to share his understanding with his peers. After giving the class a couple minutes of discussion amongst them, I will ask a two students from different groups, to go to the front of the class, and explain each step they took to find the area of a shaded part of the graph. From this presentation, the audience will either agree with their peers' conclusion, or will point out the misunderstanding that they might have done and will help them find a better approach to find the solution. By doing this, all of the students can see common mistakes that are made, and see how they can fix them. After going over this, students will notice how the line dips below x-axis when x>5, as I continue to extend the line passing through the x-axis. Students will be asked to find $\mathrm{A}(f, 0 \le x \le 7)$, as shown in figure 2.

After the student presentation, students will use their calculators to sketch on their math notebook the function $g(x) = \sqrt{16 - x^2}$. Students will be asked several questions such as:

(a) State the domain and the range of g(x).
(b) Use geometry to find $\mathrm{A}(g, 0 \le x \le 4)$.
(c) Find $\mathrm{A}(g, -4 \le x \le 4)$.

After working answering these questions, I will ask the students, "What is the relationship between the answers of (b) and (c)?" Since they will find the area of the specific

Mathematical Practices

CCSS.MATH.PRACTICE.MP1
Make sense of problems and persevere in solving them.
CCSS.MATH.PRACTICE.MP2
Reason abstractly and quantitatively.
CCSS.MATH.PRACTICE.MP8
Look for and express regularity in repeated reasoning.

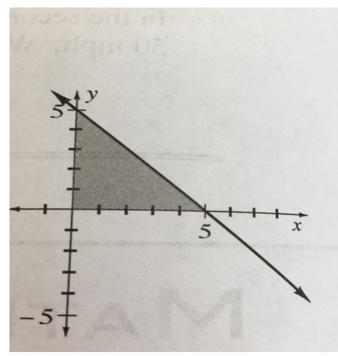

Figure 1

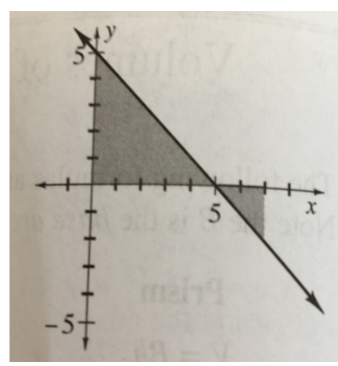

Figure 2

Benchmark Assessment 1:

(a) State the domain and the range of g(x).
(b) Use geometry to find $\mathrm{A}(g, 0 \le x \le 4)$.
(c) Find $\mathrm{A}(g, -4 \le x \le 4)$.

After working answering these

intervals provided, students will have to implement mathematical practice 1 and reason abstractly in order to find the area under the curve.

On the following day, I will provide a notes, for students to write them on their math notebooks. Then, students will describe the graph by finding both the domain and the range. While I write down the important ideas of the lesson, the student with hearing impairment will see what the I am talking about (Lewis & Doorlag, p.301, 2013). After direct instruction, I will use cooperative learning where students will practice several problems, and explain their findings to the class (Palmer, Peters, & Streetman, p.10, 2008). To ensure that the Common Core State Standard HSF.IF.A.2 is met, I will ask students to find the domain and the range of the function. For example, I will provide the function $f(x) = \frac{3}{x^2} + 1$. Students will be asked to find both domain and range. I will also provide figure 3, and ask, "Using the interval notation, state the domain and the range of each function in figure 3 (a and b)."

In addition, students will review various types of functions. Students will graph two functions on the same axis for $x \geq 4$. The two functions are $g(x) = \frac{1}{2}x + 1$ and $h(x) = -x + 6$. I will ask students to evaluate and demonstrate how both functions look like when $x = 4$. I will introduce the concept, piecewise, defined as combining parts of several functions to make a single function. Students will be introduced of how the algebraic functions of g and h can be "pieced together" as a single function. This function will be called, $f(x) = \begin{Bmatrix} \frac{1}{2}x + 1 & x < 4 \\ -x + 6 & x \geq 4 \end{Bmatrix}$ After presenting this function, students will use the mathematical practice 3 by evaluating $f(0), f(4)$, and $f(6)$ [Benchmark Assessment two]. In addition, on this same section, students will be introduced to intuitive notion of continuity. In the Chapter 2, students will learn a formal definition for continuity, but for now, a function is continuous if the graph of the function can be drawn without lifting the pencil from the paper. The students will all receive white boards and they will be told if f(x) is continuous. When students decide if the function is continuous or discontinuous, students will hold up their whiteboard so I can formatively assess my students' understanding [benchmark assessment 3]. One of the students will be asked to explain their reasoning. From

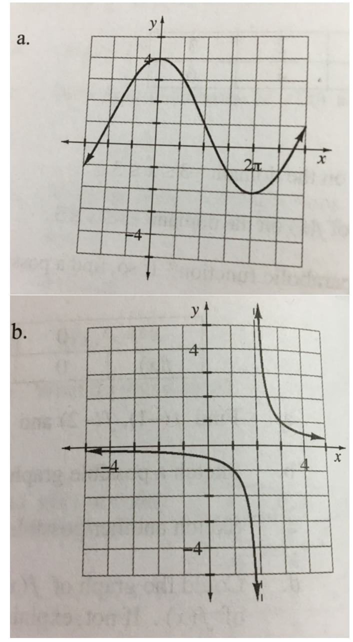

Figure 3

Benchmark Assessment 2:

Based on the function $f(x) = \begin{Bmatrix} \frac{1}{2}x + 1 & x < 4 \\ -x + 6 & x \geq 4 \end{Bmatrix}$, evaluate $f(0), f(4)$, and $f(6)$. Show your work.

Benchmark Assessment 3:

Is the function f(x) continuous or not continuous? Explain your reasoning.

this quick assessment, I will receive feedback and I will decide whether I should review back and explain each step, or move on to a more challenging problem. During this process, I should be able to pick up on any misconceptions that may arise quickly, and address them in class before they become prevalent within the class.

The third and final day of this learning progression, students will be working on finite differences and slopes. Students will have to implement mathematical practice 1 by using a table with finite differences labeled, and describing how Δy changes as x increases for the functions provided in class. I will provide two different functions of these parabolas. (a) $f(x) = 2x^2 - 3x + 1$

(b) $f(x) = -3x^2 + 6$

I will use the benchmark question four, and students will be able to demonstrate their understanding over how parabolas look like in finite differences.

In addition, since students will become familiar to finding slopes from a graph, I use a different benchmark assessment question to analyze the graphs. Students will be able to demonstrate their understanding in writing appropriate intervals and writing the correct slope value. I will include benchmark assessment five where students will turn in as an exit slip. Based on the exit slips, students will demonstrate through writing response their procedural fluency in identifying when the function is increasing or decreasing. By providing an exit slip, it will allow me to monitor the speed of the lessons and instruction. Furthermore, I will know as to what particular areas the majority of the class is struggling with. The exit slip serve be as a summative assessment of students' understanding of the concepts.

At this point, students will have just begun to work towards completing HSF.IF.B.4 and they will be ready to begin working on finding the average velocity on a position graph, velocity graph, and acceleration.

Benchmark Assessment 4:

Based on the results you found on the two parabolas, how do the parabolas change?

Benchmark Assessment 5:

For graphs a and b, state the interval where the function is increasing and decreasing.