**Learning Progression: Negative and Zero Exponents**

**High School Algebra**

**Demographics:** This learning progression takes place within a high school Algebra class of 25 students comprised of sophomores and freshman. The lesson is supplemented by material from the high school math textbook, “Algebra 2” by Holt, Rinehart, and Winston. The students have prerequisite knowledge on the basic laws of exponents. The class itself has access to Chromebooks, a whiteboard, document camera, markers, scratch paper, and calculators.

**Addressed CCSS for Mathematics:**

#### Expressions and Equations Work with radicals and integer exponents.

[CCSS.MATH.CONTENT.8.EE.A.1](http://www.corestandards.org/Math/Content/8/EE/A/1/)
Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 × 3-5 = 3-3 = 1/33 = 1/27.

**Addressed Mathematical Practices:**

#### [CCSS.MATH.PRACTICE.MP2](http://www.corestandards.org/Math/Practice/MP2/) Reason abstractly and quantitatively.

#### [CCSS.MATH.PRACTICE.MP3](http://www.corestandards.org/Math/Practice/MP3/) Construct viable arguments and critique the reasoning of others.

**Learning Progression Overview:**

The progression follows the same format as described in the textbook, “Algebra 2”. That is, the lesson will open up with a brief recap to refresh the students on the prerequisite knowledge that is required. After the review, the class will be presented with the concept of the zero exponent. Namely, the students need to understand that any real number to the zero power is equivalent to one. Succeeding this, the new material on negative exponents will be revealed to the class in conjunction with sample problems. Finally, the lesson will conclude with a brief overview on graphing exponential functions. A formative assessment will be given at the end to gauge student learning on the old material as well as the new.

**Supports:**

To provide support to students who may struggle during the learning progression, students will have access to each other to work in small groups. Through this, the students may find assistance from their peers. Peer-instruction of this nature is beneficial for reasons two-fold: For the students that have a grasp of the material, they will have the opportunity to teach their peers, which reinforces their knowledge. To further fortify student knowledge, the instructor may also provide general rules and multiplication properties of exponents in regards to the previous day’s material.

**Learning Progression:**

The lesson will open up with a recap to refresh the students on the prerequisite knowledge that is required. Four example warm-up problems will be given using the document camera **[1]**. After a majority of the class has completed the warm-up problems, students will be volunteered to provide a solution for each of the problems, to which the presenting student must give justification for their process. For instance, in the first warm-up problem, the student will be asked why the solution remains unchanged from the original problem, to which they should reply something to the notion that the variables cannot be simplified because there are no “like-terms”. After the warm-up problems have been effectively completed, the instructor will pause the lesson to do the first concept check **[2]**. In the concept check, the teacher is presenting an example solved incorrectly to which the class will be asked to identify each of the mistakes in the solution **[3]** and then provide the correct answer. At this point, the instructor will inquire to the class about how comfortable they are with the previous lesson’s material and whether they would like to do some more examples. Once the class feels adequate about the multiplication properties of positive exponents, the lesson will proceed to cover the new material on utilizing negative and zero exponents. While negative exponents can be explained as being the reciprocal of their positive counterparts, some students are likely to have difficulty in accepting the fact that any number to the zero power is equal to one. To exemplify this, the instructor will start with 33 and have the class verbally relay the answer. Then the instructor will write down 32 and have the class solve that one, and then solve for 31. The instructor will ask the class what is happening each time the power decreases to which the students should answer with the fact that the result is being divided by 3 each time. Building from this information, the instructor will have the class make note that at each descending exponent, the previous result is being divided by the base (which is 3). From here, the instructor must ask the class, if the previous number is being divided by 3, then 30 must be equal to 31 divided by 3 which results in 1 **[4]**. This useful tool is not utilized in the book *Algebra* 2, so only students who are present for class on this day will be exposed to this. If the class has trouble grasping this, the instructor will have one of the struggling

**[1]** 1. (a3)(b76)(c11)

 2. (4a2)(3a3)

 3.(3a2)3

4. (2x2y3)3 \* (x2)

**[2]** Concept Check: What two things are wrong with this solution?

(3xy4)3 = 3x3y7

**[3]**

1. Did not carry the exponent to the 3.
2. Instead of multiplying the 4 by 3, it was added.

Solution: 27x3y12

**[4]** 33 = 3 \* 3 \* 3 = **27**

 32 = 3 \* 3 = **9**

 31 = **3**

 30 = **1**

**[5]** 53 = 5 \* 5 \* 5 = **125**

 52 = 5 \* 5 = **25**

 51 = **5**

 50 = **1**

students come to the front of the class using either the whiteboard or the document to work out the same process on a different base number **[5]**. Once the class feels comfortable with the zero exponent, the educator should emphasize one final time that anything to the zero exponent is equal to one **[6]**. Continuing from here, the instructor will introduce negative exponents. Simply explained, numbers that carry negative exponents are merely the reciprocal of their positive counterparts. The instructor will follow up this definition with the general form and a small example that the instructor will walk the class through **[7]**. The class will then be given one to practice. Upon completion, the instructor will provide a more difficult example from the *Algebra 2* textbook that ties in the previous day’s material in addition to the new material with negative exponents **[8]**. When the class has had adequate time to complete the difficult example, a student who has obtained the correct answer will present their solution for the class, narrating each step that they took in their solution. After this, the second concept check will be issued, following the same process as the first one where an incorrect solution will be given and the students must identify each of the mistakes **[9]**. To check student’s critical thinking, the instructor will issue a “trick-question” of 0-2. Students who have satisfactory understanding of reciprocals will understand that 0 has no reciprocals, and therefore, 0 risen to any negative power will result in undefined **[10].** The conclusion of the progression will focus on a preview of the upcoming unit on graphing exponential functions. The instructor will address the function y = 2x and create a T-chart to where the instructor will provide the x-values and the class will verbally relay the corresponding y-values **[11]**. From here, the instructor will draw a graph that connects each of the ordered pairs and have the class appreciate how the graph is very different from the linear graphs that they have worked with previously, in that the y-values increase much faster than the x-values. For an informal trick, the instructor might inform the class that exponential functions resemble steep rollercoaster rides. The final assessment will be subsequently issued and will cover all material except for graphing exponential functions, which will be covered more and assessed in the next lesson.

**[6]** Ostrich0 = 1

**[7]** a-n = $\frac{1}{a^{n}}$ where a is not 0.

Ex) 5x-4 = $\frac{5}{x^{4}}$ because x-4 is the reciprocal of x4 which is $\frac{1}{x^{4}}$

**[8]** (22x-3)-3 = 2-6x9 = $\frac{x^{9}}{2^{6}}$ = $\frac{x^{9}}{64}$

**[9]** Concept Check: What is wrong with the following solution?

11x-4 = $\frac{1}{11x^{4}}$

The 11 does not go into the denominator. For that to happen, it would have to be 11-1 in the numerator.

Solution: $\frac{11}{x^{4}}$

**[10]** 0-2 = $\frac{1}{0^{2}}$ = undefined

**[11]** 

**Multiplication Properties of Exponents Notes**

Let **a** and **b** be real numbers and **x** and **y** be positive integers.

1) Product of Powers Property

ax \* ay = ax + y

Example) 32 \* 37 = 32 + 7 =

2) Power of a Power Property

( ax ) y = axy

Example) ( 52 )4 = 52(4) =

3) Power of a Product Property

( a \* b )x = ax \* bx

Example) ( 2 \* 3 )6 = 26 + 36 =

Name: Date:

**Simplify. Show all work.**

1. ( x4 )( x3 )
2. ( 2x-10 )-1 (x3y-1)
3. (1x3y3) -3
4. ( y-3 )( y5 )
5. ( 50) (56)
6. (00)
7. $\frac{1}{\left(2x^{-10}y^{6}\right)}$-1