High School: Algebra 2
Unit Circle

This learning progression will be applied in a high school algebra 2 classroom, normally tenth grades. The textbook that is used in this classroom is *Core Connections: Algebra 2*, chapter 7. The Common Core State Standards aligned with this learning progress are HSF.TFA.A.1, HSF.TFA.A.2, and HSF.TFA.A.3. The Mathematical Practice standards that will be used throughout this learning progression will be MP5, MP6, and MP7.

The class has twenty students who are in tenth and eleventh grade. In the class, there are three English Language Learners (ELL), whose primary language is Spanish, and two students who are on an IEP. For my ELL students, I will partner them with a student who has been given tools to peer mentor. These students can speak fluently in English but also have a proficiency in Spanish. The ELL students and their peer can discuss the mathematics in both English and Spanish to ensure language acquisition is occurring and to allow for content understanding. The lesson is open to all students and has been designed to have a low floor high ceiling format. All students will be able to learn and meet the standards at their level. For my students with IEPs I will work with each student to modify the amount of work they have to do. By having the low floor high ceiling my students with IEPs will also be working towards meeting the CCSS and mathematical practices.

Common Core State Standards:
HSF.TFA.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
HSF.TFA.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
HSF.TFA.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

Mathematical Practices:
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.

The students understand right triangles and the special properties that are associated with right triangles. The students can determine the missing side lengths and angle measurements of right triangles. The students will be building on their knowledge of right triangles and discover the properties of sine and cosine functions. The students will model the sine and cosine function while also constructing a unit circle to be used to learn the properties of special triangles. Throughout this learning progression the students will build on prior knowledge while building new understanding of trigonometric functions. Students will discover the unit circle and the properties used of right triangles to determine the angle measurements in radians. After the students have discovered the unit circle they will use the unit circle to determine the values for special triangles.

The students will be assessed in a variety of ways throughout the learning progression to determine student growth and understanding of the content. I will provide the students with a formative assessment probe at the start of the learning progression to determine where all the students are at. From the data collected a review session may occur before the start to ensure all students understand right triangles and the special properties associated with right triangles. After I have ensured all students are at the starting point to make this learning progression the most successful the students will monitor their learning through self-evaluations on exit slips. I will use the students input to determine how the start of each task will occur to ensure all students can master the subject. The self-evaluations will occur by the students answering a set of questions prior to leaving the classroom each day as an exit slip. I will also conduct my own formative assessments during each task by asking a hinge question and using the responses to determine if the students need the material presented again or if clarification needs to take place.

Task 1: Understanding Radian Measurements (HSF.TFA.A.1)

The students understand that there are 360 degrees in a circle and the start of this segment will have the students looking to understand where radians originated from. In order for students to fully understand what a radian is this task will require the students to make a radian using a circular object and paper (see Appendix A for instructions). Once the students have created a radian measure using the different circular objects the students will discuss in their small group what a radian is and will be able to explain the relationship between a radian and the radius of a circle.

Once the students understand a radian they will move on to using their knowledge of degree measurements and convert degrees to radians. Students will use their calculator to compare the degree measurement to the radian measurements. By having the students compare degree measurements and radian measurements the students will gain the understanding required to move to the next task. Students will need to understand the relation in order to construct the Unit Circle and use it to determine the values of angle measurements in radians.

Hinge question: explain the relationship between the radius of a circle and a radian.

Exit slip: Do you have any questions that you need clarified before moving on?
Define radian.

Students will work towards MP7 when developing the understanding on the relationships.

Task 2: Build the Unit Circle (HSF.TFA.A.1 and HSF.TFA.A.2)

I will provide the students with a unit circle that does not have the measurements filled in and will begin the task with asking the students to discuss in their group what they think the degree measurements are for each of the designated angles (see Appendix B for blank unit circle and a completed unit circle). Once the students have had enough time to determine the degree measurements they will move on to filling in the rest of the unit circle using all previous knowledge they have. By having the students discover the unit circle they will be able to make connections between degrees and radians and will form a deeper understanding of the unit circle and the importance. The students will need to use their prior knowledge on special right triangles and there understanding of angle measurements the coordinate of the measurement. While the students are working with their groups to complete the unit circle I will be monitoring student progress and asking the students to explain their process to determining the radian measurement as well as the associated coordinates for each measurement. Once the students have completed filling in the unit circle we will have a class discussion to go over the unit circle to determine if the class has correctly filled in the unit circle. During the class discussion students will be able to self-assess their understanding and will indicate on their exit slip how comfortable they are with the material that was presented. After this task students will be able to use strategies and prior knowledge to complete the unit circle and explain how the first quadrant values relate to the other quadrants.

Hinge question: What strategies are you using to solve the unit circle?

Exit slip: Did you accurately complete the unit circle?
Do you have any questions that need to be clarified?
How comfortable are you with radians?

Students will work towards MP5 and MP7 when completing the unit circle.

Task 3: Unit Circle and Special Triangles (HSF.TFA.A.2 and HSF.TFA.A.3)

Students will use the Unit Circle for Task 2 to solve special triangles. Students will be working with 30°-60°-90° and 45°-45°-90° triangles. Students will need to use strategies developed throughout the unit to determine the radian measurements associated with the special triangles. Students will need to give exact values for sine, cosine, and tangent equations using radian measurements. The students understanding of the unit circle will be expanded on and they will be required to apply that knowledge to questions being posed on their worksheet. Students will be responsible for recording their answers in radians and degrees being as exact as possible.

Once the students have completed the worksheet answering the questions relating to the special triangles and the unit circle the students will have completed the learning progression. From this point the students will be able to move onto modeling sine and cosine functions while also understanding how to transform the graphs to match the information given in a problem.

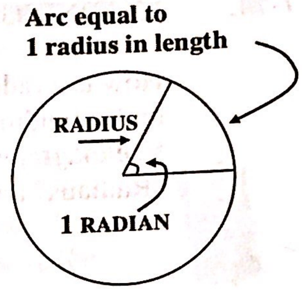
Hinge question: What is a reference angle and how are they useful?

Exit slip: Why are 30°-60°-90° and 45°-45°-90° triangles special?
What are the sine and cosine measurements of the above triangles in radians?

Students will work towards MP5, MP6, and MP7 when answering questions on their worksheet.

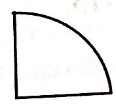
Appendix A:

7-72. HOW TO MAKE A RADIAN



Imagine wrapping the radius of a circle around the circle. The angle formed at the center of the circle that corresponds to the arc that is one radius long has a measure of exactly one **radian**.

Your teacher will provide each member of your team with a different-sized circular object and some scissors.



a. Trace your circular object onto a sheet of paper and carefully cut out the circle. Fold the paper circle in half and then in half again so that it is in the shape of a quarter circle, as in the diagram at right. How can you see the radius of your circular object in this new folded shape?

b. Place your circular object onto another sheet of paper and trace it again, only this time leave the circular object in place. Roll (or wrap) a straight edge of your folded circle around your circular object and mark one radius length on the traced circle. Then mark another radius length that begins where the first one ended. Continue marking radius lengths until you have gone around the entire circle.

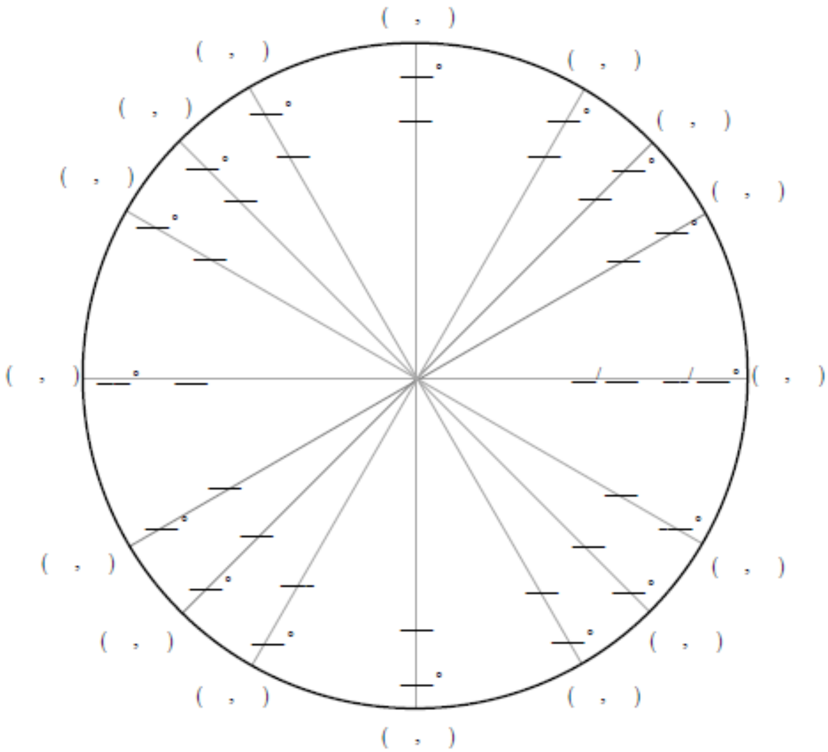
c. Remove the circular object from your paper. On your traced circle, connect each radius mark to the center, creating central angles. Each angle you see, formed by an arc with a length of one radius, measures one **radian**. Label each of the radius lengths and each angle that measures one full radian. Write a short description of how you constructed an angle with measure one radian.

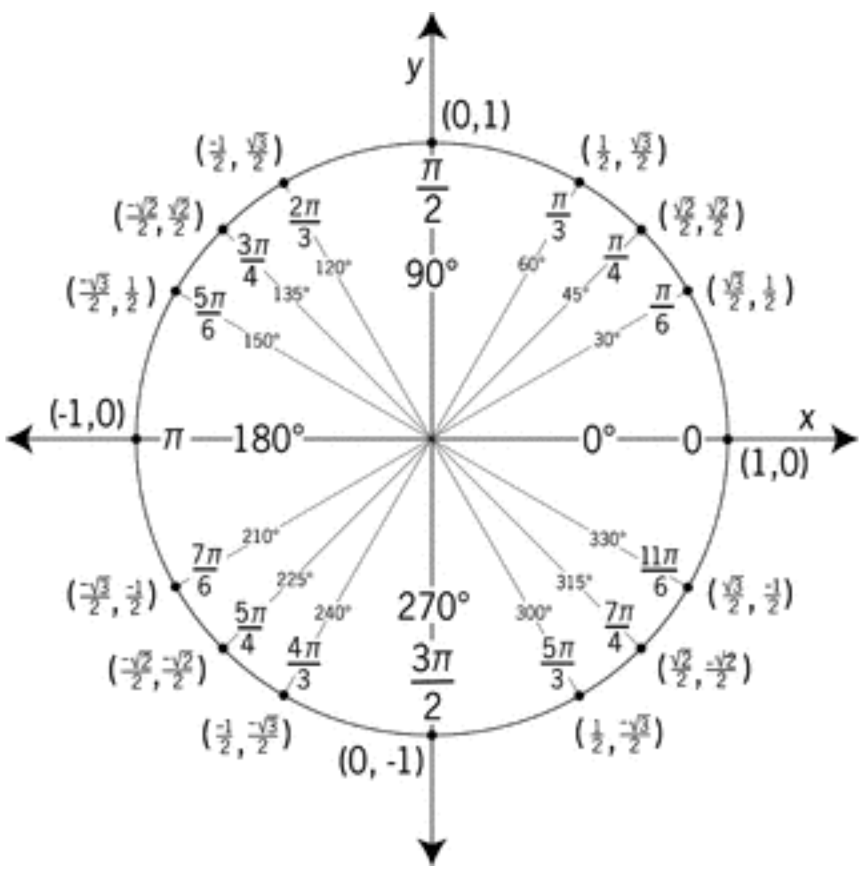
3. Assume the radius of a circle is one unit.

a. What is the area of the circle? What is its circumference?

b. How many radii would it take to wrap completely around the circle? Express your answer as a decimal approximation *and* as an exact value.

Appendix B:

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(1,0)
(-1,0)
(0, -1)
90°
180°
270°
0°
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π
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