High School Geometry

Similarity Transformations

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This learning progression will be taught in a sophomore level Geometry course at Ellensburg High School. The Common Core State Standard (CCSS) domain and cluster for this learning progression is: CCSS.MATH.CONTENT.HSG.SRT.A. There are two standards that the students will be learning: HSG.SRT.A.1 and HSG.SRT.A.2. The math practices (MP) that will be used by students during this progression will be MP1, MP3, and MP5.

The textbook used in the class is McDougall Littell's Geometry 10th edition. In teaching this learning progressions, we assume that students have a strong grasp of previous concepts required for learning similarity transformations. These concepts are HSG.CO.A.1, HSG.CO.A.2, HSG.CO.A.5, HSG.CO.B.6, and HSG.CO.C.9.

This learning progression will not be broken into separate lessons. The lessons will proceed when students have shown proficiency in the CCSS that student are being assessed on. The progression will begin with students reviewing the concept of congruence. The teacher will ask the students to provide a definition for congruence and if the students are unable to remember, the teacher will give the definition. Students will then be given one example through direct instruction from the teacher. Afterwards, a review sheet will be given to the students. The students will be able to work in groups of up to 3, but they may work alone if they wish. The review sheet will consist of several regular polygons, which are numbered, with angles and side lengths labeled. Each polygon will have a similar match as well as a congruent match on the sheet. The students will then fill in the blanks next to each polygon to identify which object it is congruent with as well as which polygon it is similar to. As students are working on the review, the teacher will walk throughout the room and formatively assess the students. At times the teacher will randomly

CCSS.MATH.CONTENT.HSG.SRT.A
Understand similarity in terms of similarity transformations

HSG.SRT.A.1
Verify experimentally the properties of dilations given by a center and a scale factor:

HSG.SRT.A.1.A
A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

HSG.SRT.A.1.B
The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

HSG.SRT.A.2
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

HSG.SRT.A.3
Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

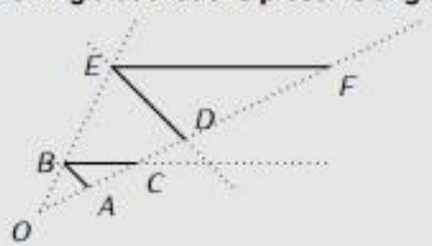
CCSS.MATH.PRACTICE.MP1
Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3
Construct viable arguments and critique the reasoning of others.

ask students why certain polygons are similar or congruent to the specific one the student picked. This will be considered a hinge question for students. If a student is struggling, and unable to explain their reasoning, the teacher will be able to give 1-on-1 instruction to attempt to clarify the concept to the student. Students will be given a similar worksheet as homework.

Once sufficient proficiency has been shown by the students, the teacher will then go back to direct instruction. The teacher will define the term dilation for the students. This definition will include the terms center as well as scale. The teacher will then explain how we dilate a line. The teacher will use a visual representation to show how to dilate a line using a ruler. The teacher will then identify the fact that these lines are parallel. The teacher will then show an example of two lines, which create an angle, being dilated. Students will be given the opportunity to conjecture on whether or not the angles are congruent. After a period of time, the teacher will go through proving the angles are congruent through the fact that the parallel lines are cut by a transversal and therefore the angles are congruent.

Showing that dilations preserve angles



The dilation with center O takes $\angle ABC$ to $\angle DEF$. The line EF is parallel to the line BC. Extending segment ED to a transversal of these two parallel lines and using the fact that alternate interior angles are congruent, we see that $\angle ABC$ is congruent to $\angle DEF$.

The teacher will then let the students conjecture on what would happen if we were to dilate a line that goes through the center of dilation. After the students are given time, the instructor will identify the behavior of line which dilate through the center.

CCSS.MATH.PRACTICE.MP5
Use appropriate tools strategically.

HSG.CO.A.1
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

HSG.CO.A.2
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

HSG.CO.A.5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.B.6
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

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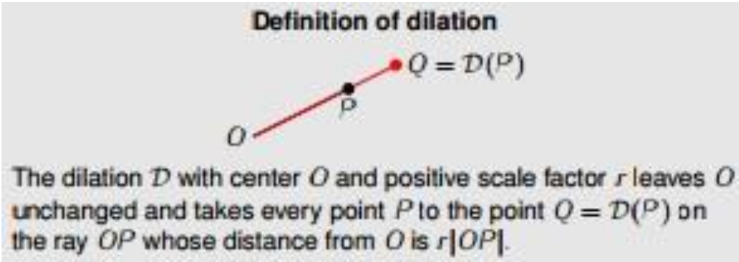


At this point, student will be given a worksheet to practice dilation. There will be 6 dilation problems. Three of the problems will deal with dilation through the center and the other three will be creating congruent angles with dilation. Students will again be allowed to work in groups of up to 3. Again the teacher will employ a similar strategy for formative assessment as the previous in-class worksheet. The hinge question for students will also be of a similar fashion. The teacher will walk through the class and ask students to explain how they dilated their lines and why they think their dilation is correct. This will be considered a hinge question for the students. At the conclusion of this in-class worksheet, a similar, yet slightly more difficult, worksheet will be handed out for homework.

Following the previous instruction, the teacher will move on to similarity transformations. This concept will be detailed through direct instruction for the students. With their proficiency in transformations in the plane as well as dilation, the students are now properly equipped for this concept. We will be combining SRT.A.2 and SRT.A.3 together to teach this concept as they are very similar. The teacher will give the students an example of how to use similarity transformations to show that two triangles are similar. The teacher will begin this process with a single triangle. The teacher will then dilate the triangle with a scale factor of their choosing. The teacher will show that the angles are congruent as shown previously. The teacher will then label the new triangle. The teacher will then show that all of the side lengths are proportional to each other. The students will be given a worksheet that has three problems similar to the example. For each problem, the students will be given a triangle and a scale. The students will then create a similar triangle using the scale and a dilation point. They will then make a proportion for each side length and show that they are

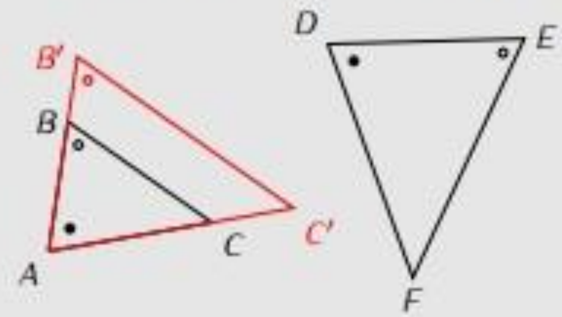
HSG.CO.C.9

Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Images from http://commoncoretools.me/wp-content/uploads/2016/03/ccss_progression_g_7_hs_2016_03_27.pdf equal to the scale. Once again the teacher will walk the room providing assistance as well as asking students to explain their work as a hinge question. Students will be given a handout for homework similar to the one completed in class.

For the final step of the learning progression, the teacher will show the Angle-Angle criterion for triangles to be similar. This will be shown to the students through direct instruction.

Proof of the AA criterion using similarity



Given $\triangle ABC$ and $\triangle DEF$ with $m\angle A = m\angle D$ and $m\angle B = m\angle E$, perform a dilation on $\triangle ABC$ with center at A so that $|AB'| = |DF|$. Because dilations preserve angles, $m\angle B' = m\angle E$, and so $\triangle AB'C'$ is congruent to $\triangle DEF$ by the ASA criterion. Since $\triangle AB'C'$ is a dilation of $\triangle ABC$, this means that $\triangle ABC$ is similar to $\triangle DEF$.

Once the example has been given by the teacher, the students will be given a culminating worksheet. This worksheet will be a homework assignment. There will be three dilation problems, two similarity transformations, and a problem to show the AA criterion for triangles to be similar. The learning progression will end on the following day with a quiz similar to the worksheet the students completed.

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