

The Learning Progression is set up to take place in an 8th/9th grade algebra class. The class will be introduced to the concept of probability using discovery-based instruction. This type of instruction has been found to be beneficial for students when scaffolding and feedback take place (Alfieri, et al., 2011). Students will be able to use probability to summarize, represent, and interpret data on a single count or measurement variable and to use their results to interpret and create linear models. The activities were obtained and modified from Math Circles, to meet standards at the secondary level. The goal for this learning progression is for students to meet and use the following standards:

- MP1: Make sense of problems and persevere in solving them
- MP4: Model with mathematics
- MP6: Attend to precision
- **Summarize, represent, and interpret data on a single count or measurement variable**
 - HSS.ID.A.1-Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **Interpret linear models**
 - HSS.ID.C.7-Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
 - HSS.IC.A.2-Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

The first lesson will consist in introducing the concept of probability by allowing students to learn by discovery. Students will be given the task to pick a horse, numbered 1-12. Then they are going to be told that there is going to be a race between the horses. The horses will move up a place if their number appears when two dices are rolled. Ex. the dices were rolled and a 2 and a 3 was obtained, then the horse with the number 5 (adding the two different values of the dices) will move up the spot. Here students will keep track of the race using a simple histogram ^{HSS.ID.A.1, MP4}

that shows their results. Once a race is over, various questions will be asked about the activity to begin developing students' thinking of probability. Students will be asked to give explanations as to why certain horses did not move as much as others, and why others did not move at all. Here students will be able to use the histogram and their own knowledge of using dice to respond to these questions. The benchmark for this day is for students to recognize the likeliness of numbers occurring, by giving statements or asking questions.

To finalize the day, students will be instructed fill out a table that shows the various ways a number can be obtained when rolling a dice. In this case the number 11 can be obtained with (5+6) and (6+5). Once students finished the table, they will be asked to rolled the dices 20 times and keep count of how many times a number comes up. From here, students will write how many times a number appeared out of the 20 times they rolled the dices. Ex. if the number 5 was rolled 7 times, they will write $\frac{7}{20}$. Students will then be asked to theorize if their findings are consistent to how many times a number is to come up, based on the table they had previously filled out.^{HSS.IC.A.2, MP4}

The second lesson of the learning progression consists on using coins to learn about probability. As a warm up activity, students will use one penny to identify the different outcomes when the penny is flipped. Then they will use two pennies to list the outcomes when both are flipped^{MP1, MP4}. Students will be required to write the probability of their results in numerical form (fraction, decimal, and percentage) and explain which outcome has higher, lower, or equal probability of occurring.

Students will be asked about the different outcomes from flipping a single penny. In students responses the teacher will be looking for answers like, $\frac{1}{2}$, 50%, 50-50. Answers should be similar to the exercises they have done; this is to confirm that they can generalize material

they have learned.^{MP1, MP4, HSS.IC.A.2} Students will then be asked to discuss what would happen if 100 pennies are tossed as a group. Students will then be given a cup with 100 pennies to find out the results.

The students will be given an activity sheet that will contain a table (Fig. 1) to be filled out during activity and a graph grid (Fig. 2) that will be use towards the end of the activity.^{MP1, MP4, MP6, HSS.ID.A.1, HSS.ID.C.7, HSS.IC.A.2} Students will be instructed to shake the cup, flip the cup and let the pennies go on the desk. Students will then separate the heads from the tails, and will write down on their table how many heads they obtained; the tails will be put back into the cup and the same process will be repeated until no pennies are left. This process will take place 3 times for students to gather a greater number of data.

The next step is for students to graph their results on the grid provided. It will be recommended to use different colors for each of the times they began the activity. Students will be showed how to do it, for then to complete the graph on their own. The benchmark for this activity is for students to create a graph based on their data, label all parts correctly and theorize if this model is consistent with the outcome from just flipping one penny.^{HSS.IC.A.2}

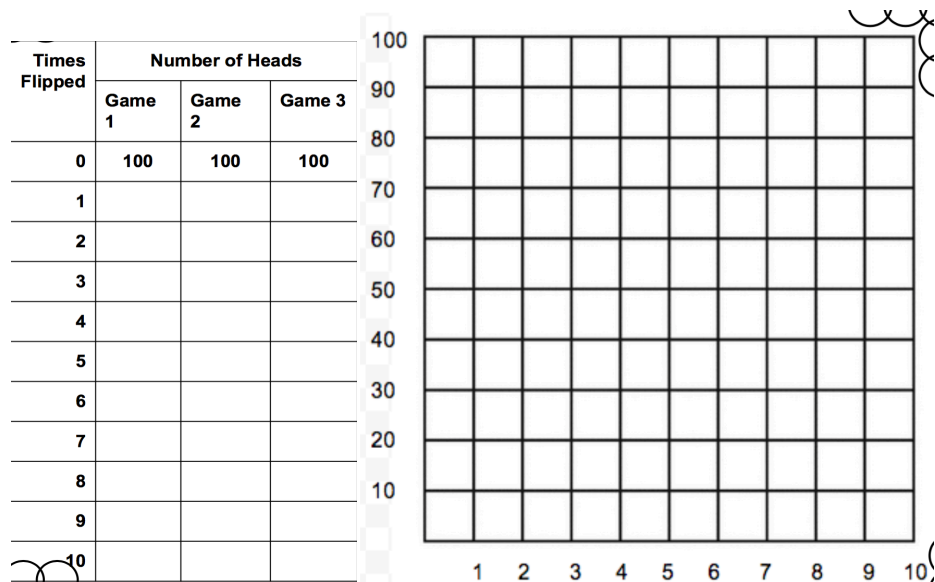


Figure 1

Figure 2

Alfieri, Louis, Brooks, Patricia J., Aldrich, Naomi J., & Tenenbaum, Harriet R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology*, 103(1), 1-18.

Kittitas Valley Math Circles