Title (primary CCSS Math with Title)

Limits of a Function – CCSS.MATH.HSF.IF.C.8

Alignment to Content Standards

CCSS.MATH.HSF.IF.C.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Tasks

At first glance, what would you expect the limit of the function to be as x approaches 2? Find the limit, if it exists. Why is predicting the values of this function at 2 difficult?

tex2html_wrap_inline125

Commentary

The purpose of this question is to test whether students understand what limits are and how to find them, given equations that don’t work out nicely. This is a beginning calculus question that takes their knowledge of limits and also their knowledge of manipulating equations and graphs. The goal is for them to realize that a limit is the number that a function approaches as the input gets closer to a specific number. This may or may not exist, or they may simply create a hole like in this problem. They will have to work with the equation in order to get the limit as x approaches a number that is actually undefined. The students will have to understand that this is not the actual point as there is a hole at this point but if it were to go through all the way, they do have the ability to calculate what the y coordinate would be.

Solution

tex2html_wrap_inline559

At first glance, the answer would appear to be zero or undefined because you don’t know what goes to zero first. Then the students are expected to manipulate the equation by factoring.

tex2html_wrap_inline561

Cancel out (x-2)

tex2html_wrap_inline567

Plug in 2:

tex2html_wrap_inline569

tex2html_wrap_inline571

The point would be (2, 11/4) but there is a hole at this point. They should explain in some sense that we created a fairly equivalent equation but it was different since we took away the hole.

