

# Learning Progression Formative Assessment

Complete this worksheet as evidence of your planning and thinking on your learning progression. This activity is intended to help you organize your learning progression and reveal information you will include when writing your learning progression. Prompts 1-3 and 5 must be completed by Jan. 10 and learning progression must be printed for class on January 17 for peer review. The revised learning progression must be submitted on Canvas by January 19. Then use feedback from peers and Dr. Oursland to revise your learning progression and post it on the Better Math blog by January 30.

1. Identify a math textbook and grade level for your learning progression

[ The grade level for my learning progression is 9<sup>th</sup> and 10<sup>th</sup> grade in high school algebra. The textbook is "Algebra I: An Integrated Approach" by Southwestern.]

2. Identify the CCSS Math domain and cluster for your learning progression

[The CCSS Math domain is High School Algebra I and we will be working on the Quantities cluster. HSN-Q.A.1-3

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.]
3. Use the CCSS Math resources (Standards, Published Learning Progression, math textbook, and web) to write an outline of math activities and benchmark assessments for each CCSS Math in the CCSS Math cluster.

[ HSN-Q.A.1

The math task will be Fuel Efficiency

<https://www.illustrativemathematics.org/content-standards/HSN/Q/A/1/tasks/930>

HSN-Q.A.2

The math task will be Giving Raises

<https://www.illustrativemathematics.org/content-standards/HSN/Q/A/2/tasks/1850>

HSN-Q.A.3

The math task will be Calories in a Sports Drink

<https://www.illustrativemathematics.org/content-standards/HSN/Q/A/3/tasks/1203>

All three of these activities were selected from Illustrative Mathematics. The task was chosen primarily because it lined up with the CCSS Math domain and cluster of this learning progression. Each CCSS in this domain had multiple options for activities. I chose the topics that would be of the most interest to the high school students in this math class. Many are starting to drive or will be driving soon so Fuel Efficiency which saves them money is of interest to them. Speaking of money, they like to have money and a task about Giving Raises is something they can relate to. The last task, Calories in a Sports Drink, is also relatable because many high school students consume sports drinks. ]

4. Write the learning progression narrative in the same format as the Published Learning Progression: The narrative is an explanation about how the conceptual understanding, procedural fluency, and math reasoning aspects of the CCSS Math will be taught in a connected way using math activities, leading questions, and benchmark assessments. The explanation should explain the purpose of the activities and how the benchmark assessment will be used in the progression of activities. Similar to the Published Learning Progression, your learning progression should have the narrative on the left – hand side and details about the math activities, benchmark assessments, and CCSS Math on the right-hand side.

## High School Algebra

Quantities and Units

Rose Myers

### Introduction:

This learning progression was designed primarily for a 9<sup>th</sup> grade algebra course. The three Common Core State Standards that this learning progression will be satisfying are from the cluster titled “Reason quantitatively and use units to solve problems,” these are HSN-Q.A.1, HSN-Q.A.2 and HSN-Q.A.3. In this course, students are focusing on mastering the Common Core State standards for Algebra. Throughout this learning progression, students will focus on three mathematical practices which are MP2, MP4, and MP6.

The curriculum these students are going through comes from the 2004 McDougal Little Algebra 1 Textbook. As our textbook was written well before Common Core State Standards were created I have chosen to cover this cluster in between our typical textbook chapters. This non-chapter unit occurs after students have completed chapter 11 which is about Rational Equations and Functions. This interlude unit serves various functions, it helps students to solidify what they have learned about rates and proportions in the previous chapter, it helps the class to satisfy all Common Core State Standards and it provides the students with real world examples of applications of units and quantities. This learning progression is supplemented not only by the previously mentioned textbook but also by activities from Illustrative Mathematics that are directly aligned to this learning progression’s associated Common Core State Standards.

I will clarify learning throughout this learning progression by providing students with a learning target aligned with each CCSS Math standard before

## Common Core State Standards

Reason quantitatively and use units to solve problems.

### CCSS.MATH.HSN-Q.A.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

### CCSS.MATH.HSN-Q.A.2

Define appropriate quantities for the purpose of descriptive modeling.

### CCSS.MATH.HSN-Q.A.3

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities

## Mathematical Practices

### **CCSS.MATH.PRACTICE.MP2**

Reason abstractly and quantitatively.

### **CCSS.MATH.PRACTICE.MP4**

Model with mathematics.

### **CCSS.MATH.PRACTICE.MP6**

Attend to Precision.

each task. This learning target will set before them the goal that is to be achieved throughout the activity. They will know because of the planning on my part that this learning target is aligned with a CCSS Math standard that they will be tested on. This will serve as motivation for them because if they are able to grasp this concept they will do better when they are assessed on it in the Smarter Balance test. Another way I am motivating student learning is through topic matter. Each task was chosen from Illustrative Mathematics because I thought it would be of interest to my students. And when they are interested in the topic they are more likely to be engaged in the activity and therefore in mathematical learning.

The central focus of this learning segment is to reason quantitatively and to use the quantities and units provided to achieve the goal of each problem. The purpose of this content is to give students the mathematical reasoning to be able to guide themselves to the desired result of mathematical questions that rely heavily on quantities and units. Ultimately the students will be able to apply these ideas in their own lives by calculating rates and doing conversions. The underlying concepts are using proportions to do unit conversions, understanding rates, and understanding variables. The simple knowledge of this learning segment is being able to do the algebra that is required in each problem. As this unit is occurring towards the end of the school year they have learned all of the algebraic procedures necessary to solve the problems.

The three main populations of students I will need to make adjustments/provide supports for are students whose native language was not English, students who have IEP's and my advanced students. A support I will provide for the first two groups of students would be a sheet of unit conversions. All students will be provided with conversions such as

## HSN-Q.A.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

### Learning Target:

I can use unit conversions and proportions to determine fuel efficiency.

### Task: Fuel Efficiency

Sadie has a cousin Nanette in Germany. Both families recently bought new cars and the two girls are comparing how fuel efficient the two cars are. Sadie tells Nanette that her family's car is getting 42 miles per gallon. Nanette has no idea how that compares to her family's car because in Germany mileage is measured differently. She tells Sadie that her family's car uses 6 liters per 100 km. Which car is more fuel efficient?

### Guiding questions:

Are the two mileages given in the same form? (no, our task says they aren't)

When we say 42 miles per gallon this is an example of a rate?

How should we set up the problem? (use proportions)

miles to kilometers and gallons to liters. However, students that need extra support will also have conversions within the metric system and within the US standard system. Such as how many feet are in a mile which I expect my other students to know. I will also provide vocabulary support via a handout, myself, or a dictionary. For my advanced students, I will provide them with some additional challenge problems to work on if they complete the activity early. I will check in with them to make sure that they accurately completed the assignment because they have the tendency to rush through their activities and make mistakes because of it. I want to ensure that they have grasped the foundational concepts before I allow them to move ahead.

The formative assessment technique I will be using will be one where I ask questions of the class so that both the students and myself know at what level of conceptual understanding, procedural fluency, and mathematical reasoning they are at. However, to encourage student voice through discourse I will have the students design the assessment questions. By having the students come up with the questions I will ask I will have a variety of options that would range from questions that students already know how to answer to questions that they have because they don't know how to answer them. Rather than only having my own questions for the class I will provide them with guidelines for their own.

My goal is that the questions the students ask will serve as hinge questions. However, in case they aren't quite what I wanted I will ask the following hinge question: "What does it mean to reason quantitatively? And in your own words what purpose do units serve when solving math problems?" These questions in conjunction get at the overarching idea of the CCSS Math for this learning progression which is "Reason quantitatively and use units to solve problems." These questions will ensure that all my students have a basic

## Formative Assessment:

Student created hinge questions. Students will be provided with the following guidelines to create questions for:

- relating to quantity
- unit conversions
- having a unit
- definition of quantity

### Required conversions:

1 Gallon = 3.78541 Liter

1 Mile = 1.60934 Kilometer

### Note:

Students may struggle with doing the reciprocal at some point. The US uses distance per volume and Europe uses volume per distance.

### Associated MP's

MP4: Model with mathematics.

MP6 Attend to precision.

This task was provided by Illustrative Mathematics, linked here:

[Fuel Efficiency](#)

understanding of what both quantity and units are before we go further in the lesson. If they don't have this foundational knowledge, I will adapt my lesson to support them in gaining this conceptual understanding so that they can later use it in procedural applications and their mathematical reasoning.

Having done the student led hinge question creating and discussion I will present the students with the following hinge question: *Choose a quantity of one unit and then choose another unit, how can you convert your initial quantity to the other unit?*. This question is a simple way to transition the students into the task for **HSN-Q.A.1**. The above support regarding the unit conversions directly corresponds to task 1 which is asking students to make is so that the two mileages are in the same units so that the girls in the story, Sadie and Nanette, can determine which car is the most fuel efficient. I specifically chose this task to be the first in our three-part learning progression for this Common Core State Standard cluster because it is something most high school students can relate to. Many students can drive or in the case of 9<sup>th</sup> graders are about to drive and having a fuel efficient car would be nice for them so that they could save money. This problem being about fuel efficiency should pique their interest and engage them. Also while there are some difficult parts of the conversion, it is achievable especially with the resources available in the class which are the conversion charts, peers and the teacher.

The task for **HSN-Q.A.2** is Giving Raises from Illustrative Mathematics. I chose this one as the second task in our sequence because I wanted a slightly challenging task in the middle of the three that would be part of this cluster. Another reason I chose it was because again the topic matter would be of interest to my students. At this age many are starting to consider getting a job because it is nice to

## HSN-Q.A.2

Define appropriate quantities for the purpose of descriptive modeling.

### Learning Target:

I can consider a situation regarding a large quantity and come up with justified ways to divide the quantity.

### Task: Giving Raises

A small company wants to give raises to their 5 employees. They have \$10,000 available to distribute. Imagine you are in charge of deciding how the raises should be determined.

- What are some variables you should consider?
- Describe mathematically different methods to distribute the raises.
- What information do you need to compute the raises for each employee?
- Make up the information you need to compute specific raises for 2 different methods and apply them to the situation. Compute the specific dollar amount each employee receives as a raise.
- Choose one of your methods that you think is most fair and construct an argument that supports your decision.

have their own money. Some students may already have jobs. Regardless of whether an individual student has or doesn't have a job they all know that getting a raise is a benefit because that means you make more money.

Prior to introducing the activity, I am going to do the formative assessment that will serve as a baseline for the class where students come up with hinge questions. I will ask the students to work in pairs to come up with different scenarios where they have a quantity of something and they need to divide it. They will then write up questions to go along with their scenarios about variables to consider, ways to divide the quantity (mathematically) and why they chose to divide it as they did. I will have 2 sets of pairs get together after their scenario is completed and they will choose which of the two scenarios to give to me. I will then go over the chosen scenarios.

At this point we will go over what the learning target is for the day and I will ease them into the Giving Raises task by stating just the initial prompt. For this activity since there are so many parts I will break it up into smaller sections with students checking in with me between each one. Each part of the task is written on a little slip of paper that I will have on my desk. The students will work in the same pairs from earlier and each time they complete a part one of the partners will come up and show me their work. If they are on the right track I will give them the slip with the next part on it. If they need assistance we can talk through their ideas and then the partner has to report back to the pair and come back to me when they have made some changes.

The variables I would like them to consider are:

- What is the salary of each employee?
- How many hours a week do they work? Are they full time or part time?
- What is their education?

**Each of the following parts (a-e) will be on slips of paper on my desk.**

- a. What are some variables you should consider?
- b. Describe mathematically different methods to distribute the raises.
- c. What information do you need to compute the raises for each employee?
- d. Make up the information you need to compute specific raises for 2 different methods and apply them to the situation. Compute the specific dollar amount each employee receives as a raise.
- e. Choose one of your methods that you think is most fair and construct an argument that supports your decision.

This task was provided by Illustrative Mathematics and can be found at the following link:

[Giving Raises](#)

- What is their position? (clerical staff, manager etc.)
- How long have they been working at the company?

However I am not expecting that they will think of all of these on their own, they need to have at least 2.

There are three main methods to distribute the raise which are:

- Method 1: Each person gets the same raise.
- Method 2: Each person gets the same percent raise based on their salary.
- Method 3: This is a slight variation on Method 1 - If some employees are part time and others are full time, every person gets the same raise per hour of work. For example, if one person works 40 hour and another person works 20 hours, then the first person gets twice as much money as the second person.

Much like with the variables I am not expecting students to come up with all of the methods I have listed. Of the three I have listed I would like them to have at least Methods 1 and 2. There may be other methods students come up with and I will evaluate those when they bring their work up to me to get the next slip.

The students response for part c are dependent on what methods the student came up with because each one needs different information as shown below.

- For Method 1 we don't need any additional information. Each person gets a \$2000 raise.
- For Method 2 we need to know the salary of each employee. Then we can figure out what percent increase will result in a total \$10,000.
- For Method 3 we need to know how much each person is working. Then we can figure out what the raise per hour will be and compute the final raises.

**Parts a, c and e show mathematical reasoning.**

a. What are some variables you should consider?

**In parts b and d students can demonstrate procedural fluency**

b. Describe mathematically different methods to distribute the raises.

c. What information do you need to compute the raises for each employee?

d. Make up the information you need to compute specific raises for 2 different methods and apply them to the situation. Compute the specific dollar amount each employee receives as a raise.

e. Choose one of your methods that you think is most fair and construct an argument that supports your decision.

**In order to answer all of the parts the students need to have some conceptual understanding but especially in parts b-e. They could probably come up with variables without understanding how to divide the quantity up.**

Students will continue working through the task in this way and at the end we will have a class discussion where we discuss which method each group thought was the fairest.

For the last standard in the cluster, **HSN-Q.A.3**, I will start the students off in a similar fashion as the other tasks. The students will be asked to come up with hinge questions based off food and beverage labels. I will have provided several labels to each table that the students can use to develop their questions. Each table has four students, at each table the four students will discuss their hinge questions and nominate one persons for me to discuss with the whole class.

After we have completed our formative assessment using the hinge questions I will introduce the Learning Target to them.

When I read the initial part of the task I will ask the students what the 16.9 ounce refers to as well as what one serving of 8 ounces and 50 calories mean. One purpose of this task is to give meaning to the quantities that students encounter on labels.

I chose this activity because many high school students, especially athletes, drink sports drinks so again it is something that the students can relate to.

The standard for this activity is teaching students how to choose an appropriate level of accuracy so in this lesson as we complete it we will be discussing as a class proper rounding. Students will see as we work through this activity as a class that depending on what we decide to round to that will make the information on the labels either correct or incorrect.

An additional resource that can be discussed if there is extra time or students express interest is the FDA website and looking at their food labeling guidelines.

Now for this task the student will start with the simple task of finding how many calories are in the full 16.9oz bottle. To do this the student must divide 16.9 by 8 (the number of ounces in a serving) and then multiply

## HSN-Q.A.3

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities

### Learning Target:

I can consider a situation regarding a large quantity and come up with justified ways to divide the quantity.

### Task: Calories in a Sports Drink

The label on a 16.9 ounce bottle of a sports drink indicates that one serving of 8 ounces contains 50 calories.

- a. Based on this information, about how many calories are in the full bottle?
- b. The label also says that the full bottle contains 120 calories. Does this agree with your estimate from part (a)? How can you explain the discrepancy (if there is a discrepancy)?
- c. The label on a 20 ounce bottle of the same sports drink says the bottle contains 130 calories. Is this consistent with the information on the 16.9 ounce bottle?

This task was provided by Illustrative Mathematics it can be found at the following link:

[Calories in a Sports Drink](#)

it by 50 which is the number of calories in one serving. The students will find that this results in the full bottle having  $105\frac{5}{8}$  calories. In part b students receive additional information about the calorie count of the sports drink which is that the full bottle contains 120 calories which is different from the answer we received from doing the math. At this point I will ask “Why do you think that our answer differs so greatly from that on the bottle?” I’m hoping that the conversations of the students will lead them to the answer that it has to do with rounding. Students can make some assumptions about how it was rounded and attempt to figure out how the company got 50 calories per serving and 120 calories in the whole bottle. The students should try rounding to the 10’s place. Once they have found something that works for part b. they can attempt that same method with the 20 ounce bottle of the same sports drink.

### **Concluding the learning progression:**

Once all associated tasks in the learning progression are completed, rather than having a test as we would for a typical chapter we will do a project for the summative assessment. The students will be told to look through newspapers and magazines and collect 2 examples of situations to be expressed algebraically. They are then supposed to come up with the algebraic expression that accompanies each. They must also write up what each variable represents, what quantities are involved and what units are being used. Students will be assigned this on a Friday and will have half the class period to begin working on it. They can then work on it over the weekend and will have one more in class work day on Monday. The assignment will be due Tuesday.

### **Best Practice: Hinge Question**

Throughout the whole Learning Progression I used the formative assessment strategy of Hinge Questions created by Dylan Wiliam. This strategy helps teachers

a. Based on this information, about how many calories are in the full bottle?

$$\begin{aligned} &(16.9 \div 8) \times 50 \\ &= 105\frac{5}{8} \text{ calories} \end{aligned}$$

b. The label also says that the full bottle contains 120 calories. Does this agree with your estimate from part (a)? How can you explain the discrepancy (if there is a discrepancy)?

c. The label on a 20 ounce bottle of the same sports drink says the bottle contains 130 calories. Is this consistent with the information on the 16.9 ounce bottle?

$$(20 \div 8) \times 50 = 125 \text{ calories}$$

This task was provided by Illustrative Mathematics it can be found at the following link:

[Calories in a Sports Drink](#)

to build a plan B into plan A. the hinge questions are specific questions that teachers can use during a lesson or task to assess students understanding of the concept before they progress to learning more. Wiliam extends the idea that having these specific thought out questions will result in higher quality responses from students that will really demonstrate their knowledge. He outlines more about this in his article "Designing Great Hinge Questions" in the September 2015 edition of the journal "Questioning for Learning".

5. Steps for planning a formative assessment process in your learning progression:
  - a. Include a formative assessment technique and questions.

[ The formative assessment technique I will be using will be one where I ask questions of the class so that both the students and myself know at what level of conceptual understanding, procedural fluency, and mathematical reasoning they are at. However, to encourage student voice through discourse I will have the students design the assessment questions. By having the students come up with the questions I will ask I will have a variety of options that would range from questions that students already know how to answer to questions that they have because they don't know how to answer them. Rather than having my own questions for the class I will provide them with guidelines for their own which are:

-relating to quantity  
-unit conversions]

- b. Explain how you will use hinge questions or tasks as a formative assessment technique to support student learning of the CCSS Math?

[ My goal is that the questions the students ask will serve as hinge questions. However, in case they aren't quite what I wanted I will ask the following hinge question: "What does it mean to reason quantitatively? And in your own words what purpose do units serve when solving math problems?" These questions in conjunction get at the overarching idea of the CCSS Math for this learning progression which is "Reason quantitatively and use units to solve problems." These questions will ensure that all my students have a basic understanding of what both quantity and units are before we go further in the lesson. If they don't have this foundational knowledge, I will adapt my lesson to support them in gaining this conceptual understanding so that they can later use it in procedural applications and their mathematical reasoning.]

- c. How will you clarify and motivate learning?

[ I will clarify learning by providing students with a learning target that aligns with each CCSS Math standard before each task. This learning target will set before them the goal that is to be achieved throughout the activity. They will know because of the planning on my part that this learning target is aligned with a CCSS Math standard that they will be tested on. This will serve as motivation for them because if they are able to grasp this concept they will do better when they are assessed on it in the Smarter Balance test. Another way I am motivating student learning is through topic matter. As I explained earlier in part 3 each task was chosen because I thought it would be of interest to my students. And when they are interested in the topic they are more likely to be engaged in the activity and therefore in mathematical learning.]

- d. What adjustments/supports will be needed for special populations of students?

[ The three main populations of students I will need to make adjustments/provide supports for are students whose native language was not English, students who have IEP's and my students who are more advanced. A support I will provide for the first two groups of students would be a sheet of unit conversions. All my students will be provided with conversions such as miles to kilometers and gallons to liters. However, my students that need extra support will also have conversions within the metric system and within the US standard system. Such as how many feet are in a mile which I would be expecting my other students to know. I will also provide vocabulary support via a handout, myself, or a dictionary. For my advanced students, I will provide them with some additional challenge problems to work on if they have completed the activity early. I will check in with them to make sure that they accurately completed the assignment because they have the tendency to rush through their activities and make mistakes because of it. I want to ensure that they have grasped the foundational concepts before I allow them to move ahead.]

- e. How will the formative assessment data be analyzed to support student learning and guide instruction (remember most formative assessment is analyzed in real time)?

[ The questions that the students are able to answer with accuracy and without much struggle are the things that the students and myself will realize they don't need extra assistance with. However, any questions that take an extensive amount of time I will know to focus on those aspects in the activities that the students will be working on. My instruction will become more focused on what they need extra support on. ]