**High School: Algebra 2**

Complex Numbers

This learning progression will be taught to a class that consists of sophomores and juniors in high school who are currently taking Algebra 2. The Common Core State Standards that will be addressed come from two different domains. The first domain is from High School: Algebra-Arithmetic with Polynomials and Rational Expressions. The CCSS Math cluster that will be addressed is “Understand the relationship between zeros and factors of polynomials.” The second domain is High School: Number and Quantity-The Complex Number System. The CCSS Math clusters that will be addressed are “Perform arithmetic operations with complex numbers,” “Represent complex numbers and their operations on the complex plane,” and “Use complex numbers in polynomial identities and equations.” The third domain is from High School: Number and Quantity-The Real Number System. The cluster that will be addressed is “Extend the properties of exponents to rational exponents.” Students will also meet Mathematical Practices 1, 2, 3, 7, and 8.

The textbook that I will use as a resource is Glencoe’s Algebra 2: Integration, Applications, and Connections. The lesson will be taught from sections eight through ten of chapter five of this book. These sections transition students from simplifying expressions including radicals and rational exponents to simplifying expressions containing numbers that are a part of the complex plane.

The central focus of this learning progression is an introduction to complex numbers and the complex plane. The progression begins with the strategies that are used in simplifying expressions involving radicals. These strategies will help student s understand how to use complex numbers and how to simplify expressions that contain complex numbers. Students will be first introduced to what a complex number is and will then learn how to graph them in the complex plane. The purpose of this learning progression is for students to gain a better conceptual understanding of the complex plane and will lead into solving quadratic equations that do not have real solutions. This progression is set up so that the entry tasks from each section review a concept or ask students to think critically about a problem that will help them understand the new information that will be taught during the lesson. How students do on this introductory information will influence where each lesson begins. This will then influence how far we get in the planned lesson and so the next day’s lesson will also be affected. Each lesson has been set up to be flexible and to run off of the previous lesson. Beginning the class with questions that lead students to recall information that they have previously learned and to explore a new way of thinking will help students be more successful during the remainder of the class period and will help students become more interested in what they are learning. Kubiszyn and Borich state in the book *Educational Testing and Measurement* that by imbedding a formative assessment into each lesson, “well-constructed performance test can serve as a reaching activity as well as an assessment. This type of assessment provides immediate feedback on how learners are performing, reinforces hands-on teaching and learning…it moves the instruction toward higher order behavior.”

**Common Core State Standards**

Extend the properties of exponents to rational exponents.

[CCSS.MATH.CONTENT.HSN.RN.A.1](http://www.corestandards.org/Math/Content/HSN/RN/A/1/)
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

[CCSS.MATH.CONTENT.HSN.RN.A.2](http://www.corestandards.org/Math/Content/HSN/RN/A/2/) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Perform arithmetic operations with complex numbers.

[CCSS.MATH.CONTENT.HSN.CN.A.1](http://www.corestandards.org/Math/Content/HSN/CN/A/1/)
Know there is a complex number i such that i2 = -1, and every complex number has the form a + bi with a and b real.

[CCSS.MATH.CONTENT.HSN.CN.A.2](http://www.corestandards.org/Math/Content/HSN/CN/A/2/)
Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

[CCSS.MATH.CONTENT.HSN.CN.A.3](http://www.corestandards.org/Math/Content/HSN/CN/A/3/)
(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

**Mathematical Practices**

[CCSS.MATH.PRACTICE.MP1](http://www.corestandards.org/Math/Practice/MP1/) Make sense of problems and persevere in solving them.

[CCSS.MATH.PRACTICE.MP2](http://www.corestandards.org/Math/Practice/MP2/) Reason abstractly and quantitatively.

[CCSS.MATH.PRACTICE.MP3](http://www.corestandards.org/Math/Practice/MP3/) Construct viable arguments and critique the reasoning of others.

[CCSS.MATH.PRACTICE.MP7](http://www.corestandards.org/Math/Practice/MP7/) Look for and make use of structure.

[CCSS.MATH.PRACTICE.MP8](http://www.corestandards.org/Math/Practice/MP8/)  Look for and express regularity in repeated reasoning.

**Lesson 1**

Lesson 1 begins with an entry task that has been designed to remind students of the process of manipulating radicals and rational exponents. These skills are necessary for the students to know and understand so that they can be successful throughout the remainder of the lesson. This entry task scaffolds the foundational information so that all of the students have seen this process before and can learn the new information with the same resources available to them. After the students have taken an appropriate amount of time to work through the entry task on their own I will collect their work and select a couple of the students’ work to show to the class. As a class, we will look for mistakes that each student may have made and will then talk about what went wrong. This is an instructional method that I have used often with these students and they see my selection of their work as a positive thing because they get to share their thoughts with their classmates and help everyone learn. After we have talked about the entry task problems the lesson begins. I will lead students through the steps involved in rationalizing an irrational denominator and the students will then work through Task 1 with a partner. After each group of two students has completed this problem they will explain each of their steps to another group of two students. Both of these activities address ccss.Math.Content.HSN.RN.A.1 and ccss.Math.Content.HSN.RN.A.2 and MP 1 and MP3. After the students work through the first part of Task 1 I will work through the second problem with the students and I will lay the foundations of imaginary solutions that a problem may have. This will lead into the following lesson. To end the lesson the students will complete the last set of problems in class and what is not completed in class will become homework.

**Tasks of Lesson 1**

**Learning Target:**

I can use the properties of exponents to simplify expressions including radicals and rational exponents.

**Entry task:**

Solve for x and leave in simplest terms:

1. $\sqrt[3]{8h^{2}}$
2. $x-4=x\sqrt{4}$

**Task 1:**

Solve for x and leave in simplest terms:

$$x-4=x\sqrt{3}$$

 $x-x\sqrt{3}=4$

 $x\left(1-\sqrt{3}\right)=4$ *Factor the GCF*

$$x=\frac{4}{1-\sqrt{3}}$$

 $x=\frac{4}{(1-\sqrt{3})}∙\frac{(1+\sqrt{3})}{(1+\sqrt{3})}$ *Rationalize the denominator*

$$x=\frac{4(1+\sqrt{3})}{1-3}$$

$$x=\frac{4+4\sqrt{3}}{-2}=-2-2\sqrt{3}$$

Solve the radical equation:

$$\sqrt{x-3}=\sqrt{2}-\sqrt{x}$$

 $(\sqrt{x-3})^{2}=(\sqrt{2}-\sqrt{x})^{2}$ *Square each side*

$$x-3=2-2\sqrt{2x}+x$$

 $-5=-2\sqrt{2x}$ *Isolate the square root*

 $(-5)^{2}=(-2\sqrt{2x})^{2}$ *Square both sides again*

$$25=8x$$



$$\frac{25}{8}=x$$

The graph of this equation never crosses the *x*-axis; therefore, there cannot be a real solution to this equation.

**Task 2:**

Solve each equation. Be sure to graph each equation using Desmos or your graphing calculator to see if your solution is possible.

1. $\sqrt{x}=3$
2. $x^{\frac{1}{2}}+4=0$
3. $7+6n\sqrt{5}=0$
4. $\sqrt{a+1}=\sqrt{a+6}-1$
5. $13-3r=r\sqrt{5}$
6. $\sqrt{h+3}+\sqrt{h-1}=5$

**Lesson 2**

Lesson 2 focuses on introducing complex numbers to the students. The lesson will begin with an entry task that makes the students think about how they learn and the steps that are involved in developing new skills in mathematics. I will ask the students to try to remember what they knew as third graders. A third grader knows how to add, subtract, multiply, and divide integers. The students will need to tell me which of the six problems included in the entry task a third grader would understand. The students should notice that a third grader would be able to solve problems 1 and potentially problem 3. Students will later learn how to divide integers and will learn how to divide an integer by another integer that the quotient will not result in an integer. My high school students will know how to square root a perfect square to solve problem 5, but problem 6 poses a foreign problem for my students. This will be their introduction to complex numbers. My students will reflect on their own learning progression and will be able to connect this new information to their past knowledge. I will then introduce my students to the notion that $\sqrt{-1}=i$. We will then talk about the connection between complex numbers and the real numbers. To complete Task 1, the students will need to use their knowledge of multiplying radicals to be able to produce a list of the powers of *i* and to see their pattern. This task will allow students to meet [CCSS.MATH.CONTENT.HSN.CN.A.1](http://www.corestandards.org/Math/Content/HSN/CN/A/1/), [CCSS.MATH.CONTENT.HSN.CN.A.2](http://www.corestandards.org/Math/Content/HSN/CN/A/2/), MP 1, and MP 2. In Task 2 I will ask students to work through some problems and to then teach their partner how to do the problems and then learn from their partner’s explanation when it is their turn. This allows the students to think through each step logically and in a way that they can explain to their peer. This allows students to meet MP 3. This activity serves the struggling students in my class. They will be set up with one of their classmates who can explain this new concept to them in a way that they can understand. This structure also gives struggling students the chance to see which step they do not understand. While trying to teach someone a student learns where their understanding is limited. Task three focuses on students learning how to graph in the complex plane. I will give students the opportunity to figure out a way to do this and will give them some of the historical background of how this concept was first approached. This will give students a deeper understanding of the importance of this new set of numbers they have to work with. I will then show students how to graph in the complex plane and will give them some practice problems. The students will work through each problem one at a time and on their own personal white board. Each student will hold up their white board after they have completed each problem and will compare their solution with other students who are holding up their boards. I will also be checking their solutions. This will give students fast feedback and will provide them the chance to clear up any misconceptions early in their learning process. This is how this lesson will conclude. This lesson may take more time than anticipated so at the place where there is another entry task a new class period may begin. This will depend of the progress of the students.

 **Tasks of Lesson 2**

**Learning Targets:**

* I know what a complex number is.
* I can graph in the complex plane.

**Entry task:**

Pretend that you only know what a third grader knows. Can you solve these problems?

1. $3+?=17$
2. $4∙ ?=24$
3. $10+?=2$
4. $4∙ ?=22$
5. $? ∙ ?=36$
6. $? ∙ ?=-36$

Now, turn to your group and discuss possible ways to solve the sixth problem.

* $i=\sqrt{-1}$
* All numbers have the form $a+bi$ where $a,b\in R$

**Task 1:**

Construct a list of the powers of *i*. Look for the pattern.

**Task 2:**

With your partner, decide who will do each set of problems. Do your set of problems. Now, with your partner, take turns explaining the steps that you took to solve each problem. When it is your partner’s turn to explain their process then it is your turn to listen to them and to ask questions when necessary.

Set 1:

1. $5+7i+3+i$
2. $2i(5+3i)$
3. $(5+7i)^{3}$
4. $2+3i^{2}$

Set 2:

1. $12+9i+7+18i$
2. $3i(5+2i)$
3. $(3+2i)^{4}$
4. $4+3i+7i^{2}$

**Task 3:**

**Entry task:**

Work with your group of four. How do you think that you could graph these complex numbers?

1. $7i$
2. $4+2i$
3. $1+9i$

Now that you know how to graph complex numbers let’s practice graphing some. On your white boards graph each of these complex numbers, one at a time; hold them up for me to check.

1. $1+6i$
2. $6+4i+8+i$
3. $i^{3}+2+3i$
4. $3i$
5. $2+1$

**Lesson 3**

Lesson 3 will begin with an entry task that reminds students of the process of combining like terms in a polynomial. The entry task gives them two examples of adding and subtracting like terms and two examples of multi-step problems that involve distributing each part of a polynomial to each part of another polynomial and then combining like terms. I then give students the opportunity to try these same strategies on complex numbers. I will then help students use what they know about rationalizing the denominator of a fraction so that they can work through Task 1. The questions that make up Task 1 are designed to make students think critically. They will need to problem-solve and will develop a deeper understanding of how complex numbers work. This task will allow students an opportunity to meet MP 7 and MP 8. Task 2 gives students the opportunity to practice what they have learned. This task will help the students become proficient in the process of combining complex numbers. I will give the students three sets of problems to work through. The students will work in their groups of four and will work through each problem on a hand-held whiteboard. Each group will need to compare their solutions with each other and will not be able to move on until all members of their group understand each step. This allows students to meet MP 3 and will provide the struggling students with the support that they need to be successful, while providing the excelling students the chance to deeply think through each step.

**Tasks of Lesson 3**

**Learning Target:**

I can use what I know about simplifying equations involving radicals and rational exponents to simplify expressions containing complex numbers.

**Entry task:**

Combine like terms:

1. $x^{5}+3x+5x^{4}+5x^{5}+2x^{2}-3x$
2. $x+4^{2}+y^{2}+x^{2}$

Distribute:

1. $\left(x+4\right)\left(x-3\right)$
2. $\left(x^{3}+2\right)\left(x-4\right)$
3. $(15+2i)(15-4i)$

**Task 1:**

1. Describe how to rationalize the denominator of $\frac{1}{a+bi}$.
2. Explain why the product of a complex number and its conjugate is always a real number.
3. Evaluate each expression if $z=1+2i$.
4. $z^{2}$
5. $\frac{1}{z}$
6. What would you think the product $z^{2}∙\frac{1}{z}$ would be without substituting $1+2i$ for *z*? Use substitution to verify your answer.

**Task 2:**

On your whiteboard:

Find the conjugates of these complex numbers.

1. $8i$
2. $4+6i$
3. $-i+1$

Find the product of each complex number and its conjugate.

1. $-10i$
2. $-6i+7$

Simplify:

1. $\frac{7}{-2i}$
2. $\frac{5+i}{1+2i}$
3. $\frac{7}{\sqrt{2}-3i}$

**Formative Assessment:**

At the end of this learning progression I will be using a formative assessment that will lead students to assess their own level of understanding. I will cover the whiteboard in the front of the room in sticky-notes. These sticky-notes will have a practice problem written on it. To solve each problem the students will need to use the information that they have learned during this learning progression. Each student will work one of the problems that are written on a post-it note and will then replace the note to the front of the room so that another student can work through the same problem. Each student will write their solution on the back of the note along with their name. After the first student has provided their solution to the problem all of the following students will need to compare their solutions. If they disagree with each other’s solutions they will need to talk through their disagreement and will need to work together to find the actual solution. The students will then repeat this process and attempt to complete as many problems as possible. During this time I will be asking students questions about their work and listening to their conversations. This will give me some idea of the concepts that we will need to review. I will also take all of the solutions from the sticky-notes and use them to create my next lesson. This assessment will be the students’ way showing me what they know and I will be able to give them feedback verbally during class.

Throughout the learning progression I will be assessing the students’ progress that has been made towards meeting the learning targets. The visual evidence that they show me and the conversations that they have in their groups will give me evidence of the level of their understanding. I will also use hinge questions to elicit evidence. I will ask students to clarify their ideas and explanations and will ask leading questions. My questions will scaffold information for students so that they can connect all of the pieces for themselves. Most of the lessons include some form of group work. This structure allows students the opportunity to think through new information and to see the parts that are still confusing them. As the students have questions, I will answer them or use their questions to guide a full class discussion during class time.