## High School Precalculus

Trigonometric functions

Rose Myers

**Introduction:**

This learning progression was designed primarily for a high school Precalculus course. The three Common Core State Standards that this learning progression will be satisfying are from the cluster titled "Extend the domain of trigonomentric functions using the unit circle," these are HSF.TF.A.1, HSF.TF.A.3, and HSF.TF.A.4. In this course, students are focusing on mastering the Common Core State standards for Functions. Throughout this learning progression, students will focus on three mathematical practices which are MP5, MP7, and MP8.

The curriculum these students are going through comes from the 2001 Precalculus with Limits A Graphing Approach by Larson, Hostetler and Edwards. This cluster is being covered in chapter 4: Trigonometric Functions. This chapter occurs after students have completed chapter 3 about Exponential and Logarithmic Functions. The students completed chapter 3 at the end of their first semester and we are starting the new semester with chapter 4. This learning progression is supplemented not only by the previously mentioned textbook but also by activities from Illustrative Mathematics that are directly aligned to this learning progression's associated Common Core State Standards.

I will clarify learning throughout this learning progression by providing students with a learning target aligned with each CCSS Math standard before each task. This learning target will set before them the goal that is to be achieved throughout the activity. They will know because of the planning on my part that this learning target is aligned with a

## Common Core State Standards

Extend the domain of trigonometric functions using the unit circle.

**CCSS.MATH.HSF.TF.A.1**

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**CCSS.MATH. HSF.TF.A.3**

Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

**CCSS.MATH.HSF.TF.A.4**

Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

## Mathematical Practices

**CCSS.MATH.PRACTICE.MP5**

Use appropriate tools strategically.

**CCSS.MATH.PRACTICE.MP7**

Look for and make use of structure.

**CCSS.MATH.PRACTICE.MP8**

Look for and express regularity in repeated reasoning

CCSS Math standard that they will be tested on. This will serve as motivation for them because if they are able to grasp this concept they will do better when they are assessed on it in the Smarter Balance test. Another way I am motivating student learning is through topic matter. Each task was chosen from Illustrative Mathematics because I thought it would be of interest to my students. And when they are interested in the topic they are more likely to be engaged in the activity and therefore in mathematical learning.

The central focus of this learning segment is to understand the connection between the trigonometric functions, special right triangles, and the unit circle.

The purpose of this content is to give students the conceptual understanding and procedural ability to be able to complete chapters 4-6 which all use trigonometric ideas. The underlying concepts are being able to perform trigonometric functions using a calculator and using the Pythagorean theorem. The simple knowledge of this learning segment is being able to do the algebra that is required in the Pythagorean theorem to solve for side lengths and knowing how to do trig functions on their calculator. Since all of the students in the class have already taken Algebra II they are familiar with the trigonometric ratios that they will be using in this chapter and they have been introduced to the special right triangles in their previous Geometry and Algebra II classes.

This class doesn't have any IEP's, 504 plans, or any students whose native language is not English. In this class in particular there are really only two main populations of students I will need to <u>make adjustments/provide supports</u> for are students who are doing the Cornerstone

Trigonometric Ratios:

$\text{Sin}\theta = \frac{opp}{hyp} = \frac{y}{r}$ $\quad$ $\text{Csc}\ \theta = \frac{hyp}{opp} = \frac{r}{y}$

$\text{Cos}\theta = \frac{adj}{hyp} = \frac{x}{r}$ $\quad$ $\text{Sec}\theta = \frac{hyp}{adj} = \frac{r}{x}$

$\text{Tan}\theta = \frac{opp}{adj} = \frac{y}{x}$ $\quad$ $\text{Cot}\theta = \frac{adj}{opp} = \frac{x}{y}$

**Note:**

Csc, Sec, and Cot are the buddies of Sin, Cos, and Tan respectively. The former are the reciprocals of the latter.

*Students will need to know these.

precalculus course, and students in the class who are struggling. A support I will provide for the Cornerstone precalculus class students would be to make sure that they are prepared so at the end of semester they will be able to pass the Cornerstone final. My struggling students, and all my students, will be made aware that there are resources of extra help available to them. There are college students after school Monday through Friday, and the teacher will be available before and after school. All students will be provided with "green sheets" that they will fill out with the degree, radian, sine, cosine, and tangents of each of the angles given via the special right triangles. This support has two benefits for the students the first being that they have to complete the sheet and the second is after completing it they will have a better understanding and can refer back to their green sheet. Students will also be provided with a notes sheet that they can fill out as we write out the notes and complete examples as a class. On this notes sheet will be included the necessary vocabulary and new concepts.

One formative assessment technique I will be using will be one where I ask questions of the class so that both the students and myself know at what level of conceptual understanding, procedural fluency, and mathematical reasoning they are at. However, to encourage student voice through discourse I will make sure to have students not only provide an answer to an example we are working through but also have them explain their thinking. By having the students explain their reasoning it will give myself and the students a chance to evaluate their understanding and discover and address any misconceptions.

My goal is that the questions the students may ask and their explanations can serve as "hinges" helping us to go deeper into the topic and improve understanding. The examples we will work through help the students to get at the overarching idea of the CCSS Math for this learning progression which is "Extend the domain of trigonometric functions using the unit circle." Doing

## Formative Assessment:

Students will do a practice green sheet quiz. They will self correct their quizzes as we go through the correct responses as a class.

Later on in the section they will do a graded green sheet quiz.

**HSF.TF.A.1**

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**Learning Target:**

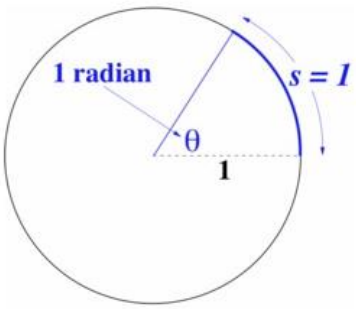
I know about angles.

I can work with radians and degrees.

**Task:** What exactly is a radian?

**Definition:**

An angle of 1 radian is defined to be the angle, in the counterclockwise direction, at the center of a unit circle which spans an arc of length 1. The picture below illustrates this definition.




**Guiding questions:**

Does this remind you of anything you have seen/done in the past? (It should remind them of unit circles and green sheet quizzes)

these examples and having students explain their thinking will ensure that all my students have a basic understanding of how to use special right triangles in conjunction with the unit circle before we go further in the lesson. If they don't have this foundational knowledge, I will adapt my lesson to support them in gaining this conceptual understanding so that they can later use it in procedural applications and their mathematical reasoning.

Another formative assessment technique will be giving the students a practice green sheet quiz that is self-corrected by going through it as a class. This prepares students for the graded green sheet quiz they will be taking a few days later. The green sheet quiz is common for all of the math teachers at EHS and is something that the students have done before in a previous math class.

Having introduced the sections learning target I will present the students with the following question: *How many radians are in a circle? Or how many degrees is a radian?.* This question is a simple way to transition the students into the task for **HSF.TF.A.1**. This task is preparing students for the above mentioned formative assessment of the green sheet quiz. They will be using a unit circle to mark off angles of specific radian measures. This will lead them into understanding the relationship between radians and degrees as well. I specifically chose this task to be the first in our three-part learning progression for this Common Core State Standard cluster because it gives a visual and hands on approach as students will either use string or a ruler to mark off the straight lines of the angles. This lesson gets students to think and while they may have some difficulty at first the procedure stays the same as they continue to mark off the radians and this will assist them in understanding it better. This task is achievable with the students prior knowledge and if need be assistance from the teacher.

The task for **HSF.TF.A.3** is Constructing a Unit Circle this is a rich math task that was specifically designed for this

What Exactly is a Radian Task continued:

a) On the unit circle below, the angles 10∘, 20∘, 30∘, etc., are indicated by black dots on the circle. Mark off angles of measure 0, 1, 2, 3, 4, 5, and 6 radians. Estimate the corresponding angle measure in degrees.

b) Estimate the angles in radians that correspond to 180∘ and 360∘.

**HSF.TF.A.3**

Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

**Learning Target:**

I know the 6 trig ratios.

I can evaluate the 6 trig functions for various angles without a calculator, find reference angles, and evaluate trig functions using a calculator.

**Task:** Constructing a Unit Circle

Materials:

-two 4"x4" squares

-scissors

-pen

-ruler

-piece of blank paper

class. Mathematics. I chose this one as the second task in our sequence because I wanted it to occur after they had a grasp of radians and degrees and how they relate to the unit circle. But I also wanted it to be early in the lesson because they will be doing the practice green sheet quiz and the real one very soon in the unit.

Prior to introducing the activity, I am going to go over the learning targets for the day. We will then start by constructing the two special right triangles that students will need to know. I will detail the steps of how to construct the 30°-60°-90° triangle. However, when it comes to labeling the side lengths I will rely on the students for what length goes with each side. Now rather than tell them how to construct a 45°-45°-90° triangle I will ask them to tell me how to form it and why their method of constructing one works. Once again, I will have them tell me the lengths of each side of this triangle. I will be getting a baseline of where the class is.

Then students will prepare their blank paper by drawing a coordinate plane on it. We will then walk through together how to construct a unit circle using these special right triangles. The goal of this assignment is that they will gain an understanding of the relation of special right triangles to the unit circle.

Once we complete the activity we will complete a notes sheet on this topic with lots of example problems for us to work on together to ensure that the students really understand it and will be able to do it on their own for their homework.

The notes will transition us into the next task that is for **HSF.TF.A.**4. This standard has to do with whether functions are even or odd.

This section will start with the introduction of the learning target. I will ask the class the following hinge question: "what can you tell me about even and odd functions?". A class discussion of ideas is welcome here.

### Constructing a Unit Circle Task continued:

Using the procedures from Pocket Protractor activity, construct a 30°-60°-90° triangle and have the students cut it out and label angles and side lengths.

Next ask students how to construct a 45°-45°-90° triangle and cut it out and label angles and side lengths

### Note:

Students will need to know these two special right triangles.

Have students draw an x and y axis on their blank piece of paper using the rulers.

Using the triangles constructed draw the angles on the unit circle and give the corresponding degrees, radians, sin, cos and tan values.

### Note:

The following tells student what is positive in each quadrant:

All Scholars take Calculus (what's positive in each quadrant)

A=All, S=Sin, T=Tan, C=Cos

S|A
T|C

The task they will be doing is from Illustrative Mathematics and is called Properties of Trigonometric Functions. They will do part a of the activity which can be seen in the side bar that I have included. After the discussion and this activity we will also address whether tan is even or odd. Then we will complete the notes sheet about this.

After completing this task the students will take their practice green sheet quiz. Will give them three angles in either radians or degrees and they have to draw the angle and write the 6 trig functions for it. This one we will correct as a class after taking it so that the students and myself know how they are doing in their understanding of the concept and their procedural ability to do it.

**Concluding the learning progression:**

Once all associated tasks in the learning progression are completed we will continue on in chapter 4. Also at this point students will take their graded green sheet quiz that they have been preparing for.

**Best Practice: Hinge Question**

Throughout the Learning Progression I used the formative assessment strategy of Hinge Questions created by Dylan Wiliam. This strategy helps teachers to build a plan B into plan A. the hinge questions are specific questions that teachers can use during a lesson or task to assess students understanding of the concept before they progress to learning more. Wiliam extends the idea that having these specific thought out questions will result in higher quality responses from students that will really demonstrate their knowledge. He outlines more about this in his article "Designing Great Hinge Questions" in the September 2015 edition of the journal "Questioning for Learning".

**CCSS.MATH.HSF.TF.A.4**

Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

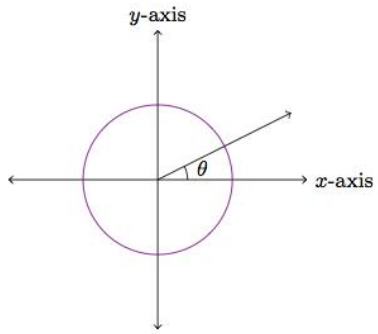
**Learning Target:**
I know whether each trigonometric function is odd or even.

**Definitions:**
-Even function f(-x)=f(x)
There is symmetry to the y-axis

-Odd Function f(-x)=-f(x)
symmetry to the origin.

**Task:** Properties of Trigonometric Functions

Below is a picture of an angle θ in the x-y plane with the unit circle sketched in purple:




a)Explain why sin(−θ)=−sinθ and cos(−θ)=cosθ. Do these equations hold for any angle θ? Explain.

Two of these tasks were provided by Illustrative Mathematics they can be found at the following links:

What exactly is a radian?

Properties of Trigonometric Functions