High School – Extended Mathematics (mixed grade level)

Base Systems and Modular Arithmetic

This learning progression is for students participating in mathematics activities outside of their standard curriculum. The group will be introduced to the concept of base systems through a study of numeric and move onto modular arithmetic by studying how the digits in place values cycle. Students will be able to understand problems and persevere in solving them, create viable arguments for their mathematical reasoning, critique the reasoning of their peers, make sense and use of mathematical structure, and make sense and use of repeating patterns. The tasks used in the progression are of my own designed. The goals of the progression is for students to develop their mathematical thinking, reasoning, and communication by completing the standards (given on column at right).

MATHEMATICAL PRACTICE STANDARDS

CCSS.MP-1 – Make sense of problems and persevere in solving them.

CCSS.MP-3 – Construct viable arguments and critique the reasoning of others.

CCSS.MP7 – Look for and make use of structure.

CCSS.MP-8 – Look for and express regularity in repeated reasoning.

One of the students has been diagnosed with Dyslexia. Observations of the student's mathematical work has given no reason to assume the student might also have Dyscalculia or Dysgraphia, as the has shown mathematical thinking and reasoning both visually and verbally. Tasks in the progression will therefore have limited reading, instructions will be given verbally, and assessment will not have a heavy focus on spelling or written grammar (although, the student will still be expected to demonstrate mathematical syntax and proper symbolic use). These accommodations are based off of a list given on *Accommodations for Students with Dyslexia*, an article posted by the International Dyslexia Association.

Another student has a 504 plan that states that the student may listen to music while working to increase focus. However, the student has used music as a focusing strategy off and on, and a clear pattern of when the student uses music has yet to be discerned. Furthermore, the student has demonstrated an eagerness to participate and develop mathematical understanding.

The final student has no known disability or a current IEP/504 plan. However, the student exhibits anti-social behaviors. The student prefers to work alone in quiet spaces, and interaction is mainly limited to non-verbal glances and blank stares. The student has also been slow to respond to questions. When asked about this, the student had said that they prefer to think carefully before speaking, and that too many questions at once is over whelming. Other meaningful conversations have also been had with the student. Based on these interactions, a wait time strategy will be implemented before instruction. This is based on a strategy in Jodi Reiss' *120 Content Strategies of English Language Learners*. Although the student is a native English speaker, this strategy still can be meaningful, as it gives students time to process information and be prepared to share. Furthermore, the student will be monitored carefully for signs of wanting to participate and calling upon the student to cooperate in the discussion.

The group of students comprise of an 8^{th} grade algebra student, a 9^{th} grade introduction to algebra student, and a 12^{th} grade geometry student. Therefore, the content covered in the progression must be selected such that all the students have entry level knowledge to participate. The content covered in this progression will be new to students, and not a part of their standard curriculums. The content presented in one task will scaffold the scheme students will need for the next task, building upon mathematical knowledge, thinking, reasoning to make conceptual connections. This strategy is based on Lev Vygotsky's Zones of Proximal Development theory. The implementation of this theory will require monitoring student progression during each task. Each task will involve students completing an inquiry activity with discussion. Each task will contain an entry question, the main inquiry, and an exit question that requires a written response. For the entry question the wait time strategy will be implemented before students respond verbally. Next, activity sheets will be passed to each student with an inquiry prompt. Students will work alone for a specified time to form ideas and questions before sharing their ideas and findings with the group. After everyone has had a chance to share their ideas, students will share a question about the activity (students will have to think of at least one question instructed by the prompt). The students will then work together to answer the question using mathematical reasoning. Questions for the students to think about and answer will also be prepared ahead of time. Finally, students will answer an exit question with a written response.

The content will cover different numerical systems, understanding base values and simple operations with them, and modular arithmetic – each task will cover each topic respectively. assessed on their mathematical thinking and reasoning either verbally or through written work. This is to accommodate both the student with Dyslexia and the student who exhibits anti-social behaviors. In this way, students will have an option of how they wish to present their mathematical thinking. Although, demonstrating mathematical thinking and reasoning verbally and visual is ideal, requiring both mediums will not be necessary to receive full points (as per the accommodation). Furthermore, since the goal of the progression is to develop the students' mathematical thinking, demonstration of conceptual understanding and procedural fluency will be presented as a major influence of students' grades. Students will also be graded on a standards-based rubric. Each task will be assessed with the same rubric (based on the standards above). Each task will also have learning targets based on content associated with the task. Students will be assessed on if they meet those learning targets, but they will weigh little to the overall grade of the task.

Task 1 – Ancient Numbers: A Study in Numerics

Summary: Students will study ancient numerical systems and compare them to the numerical system that we use today using observation, deduction, and mathematical reasoning.

Learning Targets:

- I will be able to find the value of a number in a numerical system other than the Hindu-Arabic system (i.e. I will be able to convert numbers) using deduction and mathematical reasoning.
- I will be able to recognize the differences and similarities between a numerical system other than the Hindu-Arabic system using deduction and mathematical reasoning.

Entry Questions: "What makes the number system we use good and/or bad?" A student's response might be "It's good because it has a place holder: zero."

Main Task: Students will be given an activity packet with different number systems: Egyptian, Babylonian, and Mayan. For each page, students will be given a small sample of numbers in a non-HA system and be given the prompt "You are an archeologist looking at these strange symbols for the first time. You know they must be numbers. But, can you figure out what numbers they are?" Students will work alone on deciphering the Egyptian numbers at first, then share their ideas, questions, and findings in a group discussion. I will also have a series of questions prepared for the students if discussion comes to a lull, which are as follows:

- What numbers did you figure out?
- What challenges did you find with this system?
- Did you find anything easy about understanding this system?
- Can you represent 'zero' with this system?
- How is this system like ours?
- How is this system different to ours?

Next, the students will work – at first alone, then together – to decipher the Babylonian and Mayan. This will be followed by another discussion, which can be prompted by the following questions (in addition to the previous questions):

- How are these systems like the Egyptian system?
- How are these systems different to the Egyptian system?
- How are these systems similar or different to our system?
- How do you think adding or subtracting works for these systems?
- Do any of these systems have a 'zero', or form of it?

Exit Question: "What are some key differences between some of the other number systems and ours? What are some key similarities? Why are they important?" A student's response might be "Some ancient number systems did not have place holders, which made it hard to do math.

Others made it easy to write large numbers in less space, which was good for writing a lot of information."

Assessment: Students will be assessed based on the rubrics described in the progression narrative.

Task 2 – A = 10 and 1 + 1 = 10: A Study in Base Systems

Summary: Students will study different bases systems (different than those of the task prior) and how addition and subtraction create sums and differences in those systems using observation, deduction, and mathematical reasoning.

Learning Targets:

- I will be able to compute addition and subtraction problems in base system other than base-10.
- I will be able to define the place values using a mathematical structure.
- I will be able to find connections from this task to the previous one (*Ancient Numbers*) and this one.

Entry Questions: "What are the different place values of different digits?" A student's response to this question might be "1's, 10's, and 100's."

Main Task: Students will be given an activity packet with values written in different bases and addition and subtraction problems. Each sheet will have one half with values and the other half with operations. Students will be prompted with "How are the numbers being added and subtracted?" Students will first work alone to solve the problems, then share their ideas, questions, and findings in a group discussion. I will also have a series of questions prepared for the students if discussion comes to a lull, which are as follows:

- What problems did you find easy to solve?
- What problems did you find difficult to solve?
- Are there any patterns you noticed?

Next, the students will work – at first alone, then together – to solve the rest of the problems. After some more working time, students will be asked four questions:

- What are the place values and pattern of the different systems?
- What are key characteristics of the different bases?
- Can you think about any uses for the different bases?

The students will think alone – as part of the wait time strategy – before sharing their ideas in group discussion. Scaffolding and guided assistance will be used to help the students if needed. Final responses to the last question – "Can you think about any uses for the different bases?" – should included connections to electrical computing and programing.

Finally, before the exit question, students will be asked "How does this activity relate to the previous activity?"

Exit Question: "What makes doing math with these bases easy or difficult? Why is understanding these bases so important? How does this activity relate to the previous activity?" A student's response might be "It is hard to do math in these bases because it's hard to remember what the different letters mean when I need to carry. It is important to understand these bases because computers use them. This activity relates to the previous activity because the ancient numbers used different base systems."

Assessment: Students will be assessed based on the rubrics described in the progression narrative.

Task 3 – Now 5 is Equivalent to 3, but Only Sometimes: A Study in Modular Arithmetic

Summary:

Learning Targets:

- I will be able to compute the output of a value given $mod\ n$ mathematically or with a visual.
- I will be able to find the sums and differences of modular arithmetic problems.
- I will be able to find connection from the previous two tasks (*Ancient Numbers*; *A = 10 and 1 + 1 = 10*) and this one.

Entry Questions: "Does $11 + 2 = 1$? When, and why?" A student's response might be "When you add 2 hours to 11am, you get 1pm."

Main Task: Students will be given an activity packet with modular expressions and modular arithmetic. The first sheet will have modular expressions, half shown with their equivalent values (e.g. $5\ mod\ 4 \equiv 1$) and half without. Students will work alone on the first sheet to find patterns, structure, and equivalent values of the modular expressions, then share their ideas, questions, and findings in group discussion. I will also have a series of questions prepared for the students if discussion comes to a lull, which are as follows:

- What equivalent values did you find?
- What values were difficult to find?
- What patterns to you see?
- What do you think mod does?

Next, students will work – at first alone, then together – to solve the modular arithmetic addition and subtraction problems in the remaining pages of the activity packet. After some more work time, students will asked to explain the process they have been using to solve the modular arithmetic problems. Students can do this either verbally, symbolically, or with a visual.

Exit Question: “If you had to explain to someone how to solve for any modular problem, who would you describe it? How does this activity relate to the last two activities?”

Assessment: Students will be assessed based on the rubrics described in the progression narrative.