**High School: Mathematical Decision Making**

Statistical Reasoning

This learning progression will be taught to a class that consists of juniors and seniors in high school who are currently taking a college course that is taught at their high school, Math 102: Mathematical Decision Making. The Common Core State Standards (CCSS) that will be addressed come from one domain, High School: Statistics and Probability. The CCSS clusters that will be addressed are “Make inferences and justify conclusion from sample surveys, experiments, and observational studies” and “Use probability to evaluate outcomes of decisions.” Students will also meet Mathematical Practices 3, 5, and 7.

The textbook that I will use as a resource is Pearson’s *Using and Understanding Mathematics: A Quantitative Reasoning Approach (fifth edition).* This learning progression focuses on the first two sections of chapter five from this book. These two sections introduce statistical reasoning to students and some of the vocabulary that goes along with the foundational ideas of statistics.

The central focus of this learning progression is on an introduction to statistical reasoning and the aspects of a study that may produce biased results. The progression begins with an overview of what statistical studies are and the process that someone must go through to create one. These foundational concepts will give the background knowledge that they will need to use throughout the remainder of the learning progression. The students will then learn about some different types of studies and how to avoid bias while creating a study. In the third task of this progression, the students will use all of these ideas to find their own methods of testing the validity of a study’s results. At the end of the progression, the students will be asked to design their own study and will work on conducting this study as a project that will be added on to over time. After completing this learning progression the students will learn how to analyze data and how to represent it graphically to be able to use the results of a study to make decisions with. The beginning steps of this project will be the students’ assessment.

**Common core state standards**

Make inferences and justify conclusion from sample surveys, experiments, and observational studies

[CCSS.MATH.CONTENT.HSS.IC.B.3](http://www.corestandards.org/Math/Content/HSS/IC/B/3/)
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Use probability to evaluate outcomes of decisions

[CCSS.MATH.CONTENT.HSS.MD.B.6](http://www.corestandards.org/Math/Content/HSS/MD/B/6/)
Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

[CCSS.MATH.CONTENT.HSS.MD.B.7](http://www.corestandards.org/Math/Content/HSS/MD/B/7/)
Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

**Mathematical Practices**

[CCSS.MATH.PRACTICE.MP3](http://www.corestandards.org/Math/Practice/MP3/) Construct viable arguments and critique the reasoning of others.

[CCSS.MATH.PRACTICE.MP5](http://www.corestandards.org/Math/Practice/MP5/) Use appropriate tools strategically.

[CCSS.MATH.PRACTICE.MP7](http://www.corestandards.org/Math/Practice/MP7/) Look for and make use of structure.

**Task 1**

Lesson 1 begins with and entry task that requires students to assess their understanding of a statistical study. This task is connected to [CCSS.MATH.CONTENT.HSS.IC.B.3](http://www.corestandards.org/Math/Content/HSS/IC/B/3/), MP3, MP 5, and MP 7.

**Learning Target:**

I know what a statistical study is.

I can create a statistical study.

**Entry task:**

Is it safe to use a cell phone while driving? The science of statistics provides a way to approach this question, and the results of many studies now suggest that the answer is no. A study by the Harvard Center for Risk Analysis, for example, found that cell phone use(either talking or testing) is involved in about 6% of all automobile crashes in the United States—which adds up to more than 600,000 crashes, 300,000 injuries, and nearly 3000 deaths each year. Work in groups of two to research the issues raised in the following questions. Discuss your findings.

1. Think about the physical process of using a cell phone while driving (either talking or texting), and list possible reasons why it could be distracting and cause accidents.
2. When the link between cell phone use and accidents was first discovered, many people thought the problem could be solved mandating that only hand-free cell phone systems be allowed in cars, and many localities, states, and nations enacted laws allowing drivers to use only hands-free systems. However, more recent studies show that hands-free systems are nearly as dangerous as regular cell phones—and that talking on a hands-free system is much more dangerous than talking to a passenger sitting next to you. Why don’t hands-free systems eliminate the danger of cell phone use while driving?
3. The fact that many accidents involve cell phone use does not necessarily prove that the use of cell phones caused the accidents. What kinds of studies might prove that cell phone use in the cause of accidents? How could such studies be conducted? Look for results of actual studies of this issue.
4. Statistical studies are most useful when they lead to intelligent action. Given the apparent link between cell phone use and driving, what do you think should be done about the issue? Defend your opinion.

**Task 1:**

Work with your partner:

For each of the following cases, describe the population, sample, population parameters, and sample statistics.

1. Agricultural inspectors for Jefferson County measure the levels of residue form three common pesticides on 25 ears of corn from each of the 104 corn-producing farms in the country.
2. Anthropologists determine the average brain size of early Neanderthals in Europe by studying skulls found at three sites in southern Europe.

**Definitions:**

In a statistical study …

* Population- the complete set of people of things being studied
* Sample-the subset of the population from which the raw data are actually obtained
* Population parameters-specific characteristics of the population that a statistical study is designed to estimate
* Sample statistics-numbers or observations that summarize the raw data

Introduce the process of designing a statistical study to students using this list of steps.

Basic steps in a statistical study:

1. State the goal of your study precisely. That is, determine the population you want to study and exactly what you would like to learn about it.
2. Choose a representative sample form the population.
3. Collect raw data from the sample and summarize these data by finding sample statistics of interest.
4. Use the sample statistic to infer the population parameters.
5. Draw conclusions: Determine what you learned and whether you achieved your goal.

**Task 2**

This task is connected to [CCSS.MATH.CONTENT.HSS.IC.B.3](http://www.corestandards.org/Math/Content/HSS/IC/B/3/), MP 3, and MP 5.

**Learning Targets:**

I know what a bias is and how it affects statistical studies.

I know about different types of experiments.

**Entry Task:**

For each of the following cases, describe the population, sample, population parameters, and sample statistics.

1. Astronomers typically determine the distance to a galaxy (a huge collection of billions of stars) by measuring the distances to just a few stars within it and taking the mean (average) of these distance measurements.
2. A Gallup poll of 1051 American adults shows that 32% of Americans say they have been spending less in recent months and 27% say they are saving more now and intend to make this their new, normal pattern in the years ahead.

After the students complete the entry task, they will take notes on:

Common Sampling Methods:

* Simple random sampling: We choose a sample of items in such a way that every sample of a given size has an equal chance of being selected.
* Systematic sampling: We use a simple system to choose the sample, such as selectin g every 10th or every 50th member of the population.
* Convenience sampling: We use a sample that is convenient to select, such as people who happen to be in the same classroom.
* Stratified sampling: We use this method when we are concerned about differences among subgroups and then draw a simple random sample within each subgroup. The total sample consists of a all the samples from the individual subgroups.

Two basic types of statistical study:

1. Observational study-researchers observe or measure characteristics of the sample members but do not attempt to influence or modify these characteristics.
2. Experiment-researchers apply a treatment to some of all of the sample members and then look to see whether the treatment has any effects.

Blinding in Experiments:

* Single-blind experiment- the participants do not know whether they are members of the treatment group or members of the control group, but the experimenter do know.

Double-blind experiment-neither the participants nor the experimenters know who belongs to the treatment group and who belongs to the control group.

Vocabulary:

* Treatment group-the group of sample members who receive the treatment being tested in an experiment. (Must be selected randomly and be alike in all respects except for the treatment.)
* Control group-the group of sample members who do not receive the treatment being tested in an experiment. (Must be selected randomly and be alike in all respects except for the treatment.)
* Placebo-lacks the active ingredient of a treatment being tested in a study, but is identical in appearance to the treatment. Thus, study participants cannot distinguish the placebo from the real treatment.
* Placebo effect-the situation in which patients improve simply because they believe they are receiving a useful treatment.
* Case-control study is an observational study that resembles an experiment because the sample naturally divided into two (or more) groups. The participants who engage in the behavior under study form the cases, which make them like a treatment group in an experiment. The participants who do not engage in the behavior are the controls, making them like a control group in an experiment.

**Activity:**

Use the information from today’s notes to design your own studies. Your study must include a population, sample, population parameters, and the sample statistic. You must also describe the type of sampling that you are using and method of sampling that you are using. With a partner, design two realistic studies that a company or organization could potentially conduct.

**Task 3**

This task is connected to [CCSS.MATH.CONTENT.HSS.MD.B.6](http://www.corestandards.org/Math/Content/HSS/MD/B/6/), [CCSS.MATH.CONTENT.HSS.MD.B.7](http://www.corestandards.org/Math/Content/HSS/MD/B/7/), MP 3, and MP 7.

**Learning Targets:**

I can identify sources of error in a statistical study to test its validity.

I can share my ideas with others in a way that gives sufficient proof of my claims.

**Entry- task:**

For the following studies, describe the population, sample, population parameters, and the sample statistic.

1. In order to gauge public opinion on how to handle Iran’s growing nuclear program, the Pew Research Center surveyed 101 Americans by telephone.
2. An AP/CBS telephone poll of 998 randomly selected Americans revealed that 6 in 10 people believe there has been progress in finding a cure for cancer in the last 30 years.

**Activity for Group Work:**

Read each statistical study and try to identify a source of error. After you have found a particular source of error, try to describe this source in a way that can be used in other studies.

1. A newspaper reports: “Researchers gave each of the 100 participants their astrological horoscopes, and asked them whether the horoscopes appeared to be accurate. Eighty five percent of the participants reported that the horoscopes were accurate. The researchers concluded that horoscopes are valid most of the time.”
2. By 1963, enough research on the health dangers of smoking had accumulated that the Surgeon General of the United States publicly announced that smoking is bad for health. Research done since that time has built further support for this claim. However, while the vast majority of studies show that smoking is unhealthy, a few studies found no dangers from smoking, and perhaps even health benefits. These studies generally were carried out by the Tobacco Research Institute, funded by the tobacco companies.
3. The television show *Nightline* conducted a poll in which viewers were asked whether the United Nations headquarters should be kept in the United States. Viewers could respond to the poll by paying 50 cents to call a “900” phone number with their opinions. The poll drew 186,000 responses, of which 67% favored moving the United Nations out of the United States. Around the same time, a poll using simple random sampling of 500 people found that 72% wanted the United Nations to stay in the United States. Which poll is more likely to be representative of the general opinions of Americans?
4. A Roper poll reported in *USA Today* involved a survey of the wealthiest 1% of Americans. The survey found that these people would pay an average of $487,000 for “true love,” $407,000 for “great intellect,” $285,000 for “talent,” and $259,000 for “eternal youth.”

1. Law enforcement authorities try to stop illegal drugs from entering the country. A commonly quoted statistic is that they succeed in stopping only about 10% to 20% of the drugs entering the United States. Should you believe this statistic?
2. Radon is a radioactive gas produced by natural processes (the decay of uranium) in the ground. The gas can leach into buildings through the foundation and can accumulate in relatively high concentrations if doors and windows are closed. Imagine a study that seeks to determine whether radon gas causes lung cancer by comparing the lung cancer rate in Colorado, where radon gas is fairly common, with the lung cancer rate in Hong Kong, where radon gas is less common. Suppose the study finds that the lung cancer rates are nearly the same. Is it fair to conclude that radon is not a significant cause of lung cancer?
3. The Republican national Committee commissioned a poll to find our whether Americans supported a tax-cut proposal. Asked whether they favored the tax cut, 67% of respondents answered yes. Should we conclude that Americans supported the proposal?
4. The school board in Boulder, Colorado, created a hubbub when it announced that 28% of Boulder school children were reading “below grade level,” and hence concluded that methods of teaching reading needed to be changed. The announcement was based on reading tests on which 28% of Boulder school children scored below the national average for their grade. Do these data support the board’s conclusion?
5. An experiment is conducted in which the weight losses of people who try a new “Fast Diet Supplement” are compared to the weight losses of a control group of people who try to lose weight in other ways. After eight weeks, the results show that the treatment group lost an average of one half pound more than the control group. Assuming that it has no dangerous side effects, does this study suggest that the Fast Diet Supplement is a good treatment for people wanting to lose weight?

**Formative Assessment:**

At the end of this learning progression, I will ask the students to design their own study. At this point, the students know how to design the basic aspects of a study and they know how to ask reasonable questions. The students also know different methods that are used in conducting the study. This assessment is designed to give students the opportunity to apply what they know to produce something that interests them. The students will need to ask a question that they would like to answer and then design a study that will help them answer the question. In later lesson students will learn how to use different types of graphs and methods of analyzing data so that they can do something with the data that they will gather, but for now they only need to come up with the basic ideas. If students can be successful in designing a quality study, then I will know that they have met the learning objectives.